DEVELOPMENT OF THE WAVE METHOD OF CALCULATION OPERATIONAL PARAMETERS OF A DOUBLE-CIRCUIT POWER TRANSMISSION LINE WITH KNOWN VALUES OF OUTPUT VOLTAGES AND CURRENTS AT THE END OF THE LINE

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Abstract: Electrical energy from the place of its generation is transmitted to consumers of various capacities. The distance from the source of electrical energy to the consumer can vary from several meters to several thousand kilometers. In this regard, the accurate determination of the operating parameters of the power transmission line (PTL) is a mandatory and necessary condition for the PTL normal functioning. In the current-carrying parts of the double-circuit PTL there are six incident and six reflected waves of the electromagnetic field. They determine voltages and currents. A scheme is proposed for the distribution of these waves along linear wires of a homogeneous section of a double-circuit PTL. This scheme shows that the current-carrying parts of the adjacent wires have a significant impact on voltages and currents in one wire. This scheme illustrates the distribution of the amplitude values of electromagnetic field waves, defined as the integration constant. Using the integration constants, the propagation constants of electromagnetic waves along the linear wires of the PTL and the corresponding wave impedances, one can obtain the amplitude values of the incident and reflected waves at any point of the double-circuit PTL, and hence the currents and voltages in the double-circuit PTL. The article presents a method for determining the currents and voltages in a double-circuit PTL according to the load. The proposed method will allow determining the qualitative and quantitative indicators of electrical energy (induced voltage) appearing from each wire separately and provide the possibility of their elimination, which will improve the quality of electrical energy.

Keywords: development of methodology for calculation the operating parameters of a double circuit power transmission line; incident and reflected waves; self and mutual wave impedances.

РАЗРАБОТКА ВОЛНОВОЙ МЕТОДИКИ РАСЧЕТА РЕЖИМНЫХ ПАРАМЕТРОВ ДВУХЦЕПНОЙ ЛИНИИ ЭЛЕКТРОПЕРЕДАЧИ ПРИ ИЗВЕСТНЫХ ВЫХОДНЫХ ЗНАЧЕНИЯХ НАПРЯЖЕНИЙ И ТОКОВ В КОНЦЕ ЛИНИИ

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Резюме: *Электрическая энергия от места ее генерации передается потребителям различной мощности. Расстояние от источника электрической энергии до потребителя может быть от нескольких метров до нескольких тысяч километров. В связи с этим точное определение режимных параметров линии электропередачи (ЛЭП) является обязательным и необходимым условием для нормального функционирования ЛЭП. В токоведущих частях двухцепной ЛЭП присутствуют шесть падающих и шесть отраженных волн электромагнитного поля. Именно они определяют значения напряжений и токов. Для учета взаимного влияния проводов в статье предложена схема распределения этих волн по линейным проводам однородного участка двухцепной ЛЭП. Из этой схемы видно, что на величину напряжений и токов в одном проводе оказывает существенное влияние токоведущие части соседних проводов. Эта схема иллюстрирует распределение амплитудных значений волн электромагнитного поля, которые являются постоянными интегрирования. С помощью постоянных интегрирования, постоянных распространения электромагнитных волн по линейным проводам ЛЭП и соответствующих волновых сопротивлений можно получить наглядное представление об амплитудных значениях падающих и отраженных волн в любой точке двухцепной ЛЭП, а значит, и о токах и напряжениях в двухцепной ЛЭП. В статье представлена методика определения токов и напряжений в двухцепной ЛЭП по нагрузке. Предлагаемая методика позволит определять качественные и количественные показатели электрической энергии (наведенное напряжение), появляющиеся от каждого провода в отдельности и обеспечит возможность их устранения, что позволит повысить качество электрической энергии.*

Ключевые слова: *разработка методики расчета режимных параметров двухцепной ЛЭП; падающая и отраженная волна; собственные и взаимные волновые сопротивления.*

Introduction

Usually, electrical energy is delivered from power plants where it is generated, to consumers using the overhead power transmission lines (PTL), which are characterized by transmission capacity, length and design.

Most consumers of high power are located far from the places of electricity generation. So, the transportation of electrical energy has to be carried out by overhead power lines of voltage of 35 kV and higher. Often such PTL are double-circuit. The use of double-circuit power transmission lines allows one to save raw materials (material for new supports), reduce the exclusion zone, improve the environmental situation, etc.

Electricity supplying organizations are obliged to provide consumers with high-quality electrical energy that meets the requirements of current regulatory documents[1]. For load planning, it is necessary to calculate the modes for each PTL. In this case, it is advisable to take into account the largest number of factors affecting the transmission of electrical energy along power lines.

The calculation of electric networks modes is crucial for electric power systems. One of the main tasks is to calculate the modes of double-circuit overhead power lines, in particular, to determine currents and voltages at any point on the power line wires. To solve this task it is necessary to know voltages and currents either at the beginning of the power line or at its end, in addition to the primary parameters of the analyzed PTL.

The article provides an option for calculation operational parameters at known voltages and currents at the end of power lines. This information can be obtained at the substation under consideration.

Experimental technique

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[1] GOST 32144-2013. Electric Energy. Electromagnetic compatibility of technical equipment. Standards of quality of electrical energy in general-purpose power supply systems. Moscow: Standardinform, 2014. 16 p.

The transmission of electrical energy is accompanied by directional distribution of electromagnetic field [1], which propagates through an unloaded power transmission line according to harmonic laws [2].

Typically, analysis of transmission of high-quality electrical energy is performed for a single line wire. This is true, since the process of such energy transmission through each linear wire is the same. In this case, the electromagnetic connections between the PTL linear wires are so insignificant that taking them into account does not make sense [3, 4]. But this is true only for high-quality electrical energy. Otherwise, these electromagnetic interconnections should be taken into account.

Electricity is transported through each linear wire of a homogeneous section of a double-circuit power transmission line by six incident waves of the electromagnetic field and six reflected waves [1]. This fact is confirmed by the equations of distribution of phase voltage and linear current [5]. For a line wire, they have the following form:

$$\dot{U}_{A'} = \frac{1}{6}\sum_{i=1}^{6}\left(A_{A'(2i-1)}e^{\gamma_i l} + A_{A'2i}e^{-\gamma_i l} \right), \tag{1}$$

$$\dot{I}_{A'} = \frac{1}{6}\left[\sum_{i=1}^{6}\left(\frac{A_{A'2i}e^{-\gamma_i l}}{\underline{Z}_{cA'i}} - \frac{A_{A'(2i-1)}e^{\gamma_i l}}{\underline{Z}_{cA'i}} + \frac{A_{B'2i}e^{-\gamma_i l}}{\underline{Z}_{cA'B'i}} - \frac{A_{B'(2i-1)}e^{\gamma_i l}}{\underline{Z}_{cA'B'i}} + \frac{A_{C'2i}e^{-\gamma_i l}}{\underline{Z}_{cC'A'i}} - \frac{A_{C'(2i-1)}e^{\gamma_i l}}{\underline{Z}_{cC'A'i}} + \right.$$

$$\left. + \frac{A_{A''2i}e^{-\gamma_i l}}{\underline{Z}_{cA'A''i}} - \frac{A_{A''(2i-1)}e^{\gamma_i l}}{\underline{Z}_{cA'A''i}} + \frac{A_{B''2i}e^{-\gamma_i l}}{\underline{Z}_{cA'B''i}} - \frac{A_{B''(2i-1)}e^{\gamma_i l}}{\underline{Z}_{cA'B''i}} + \frac{A_{C''2i}e^{-\gamma_i l}}{\underline{Z}_{cC''A'i}} - \frac{A_{C''(2i-1)}e^{\gamma_i l}}{\underline{Z}_{cC''A'i}} \right) \right], \tag{2}$$

where $A_{A'2i}$ and $A_{A'(2i-1)}$, $A_{B'2i}$ and $A_{B'(2i-1)}$, $A_{C'2i}$ and $A_{C'(2i-1)}$, $A_{A''2i}$ and $A_{A''(2i-1)}$, $A_{B''2i}$ and $A_{B''(2i-1)}$, $A_{C''2i}$ and $A_{C''(2i-1)}$ are the integration constants for the *i-th* pair of electromagnetic field waves; γ_i is the propagation constant for the same pair of electromagnetic field waves; $\underline{Z}_{cA'i}$, $\underline{Z}_{cB'i}$, $\underline{Z}_{cC'i}$, $\underline{Z}_{cA''i}$, $\underline{Z}_{cB''i}$, $\underline{Z}_{cC''i}$ are the self-wave impedances; $\underline{Z}_{cA'B'i}$, $\underline{Z}_{cC'A'i}$, $\underline{Z}_{cA'A''i}$, $\underline{Z}_{cA'B''i}$, $\underline{Z}_{cC''A'i}$, $\underline{Z}_{cB'C'i}$, $\underline{Z}_{cB'A''i}$, $\underline{Z}_{cB'B''i}$, $\underline{Z}_{cB'C''i}$, $\underline{Z}_{cC''A''i}$, $\underline{Z}_{cA''B''i}$, $\underline{Z}_{cC''A''i}$ are the mutual wave impedances.

The distribution of phase voltages and linear currents in other linear wires is described in a similar way.

Equations (1) and (2) indicate the presence of a double-circuit design of six incident and six reflected waves of an electromagnetic field in each linear wire of a power transmission line. Moreover, one incident and one reflected waves of them can be considered as normal waves of the considered linear wire. The remaining ten waves in this case should be considered as induced from neighboring linear wires. All this is considered in the distribution scheme of the electromagnetic field wave amplitudes along the linear wire of a homogeneous section of a double-circuit power transmission line, shown in Fig. 1. The following notation is used in the scheme: *1* and *2* are the normal incident and reflected waves of the electromagnetic field; *3, 5, 7, 9,* and *11* are the incident waves of electromagnetic field induced from neighboring wires; *4, 6, 8, 10* and *12* are the reflected waves of the electromagnetic field induced from adjacent wires.
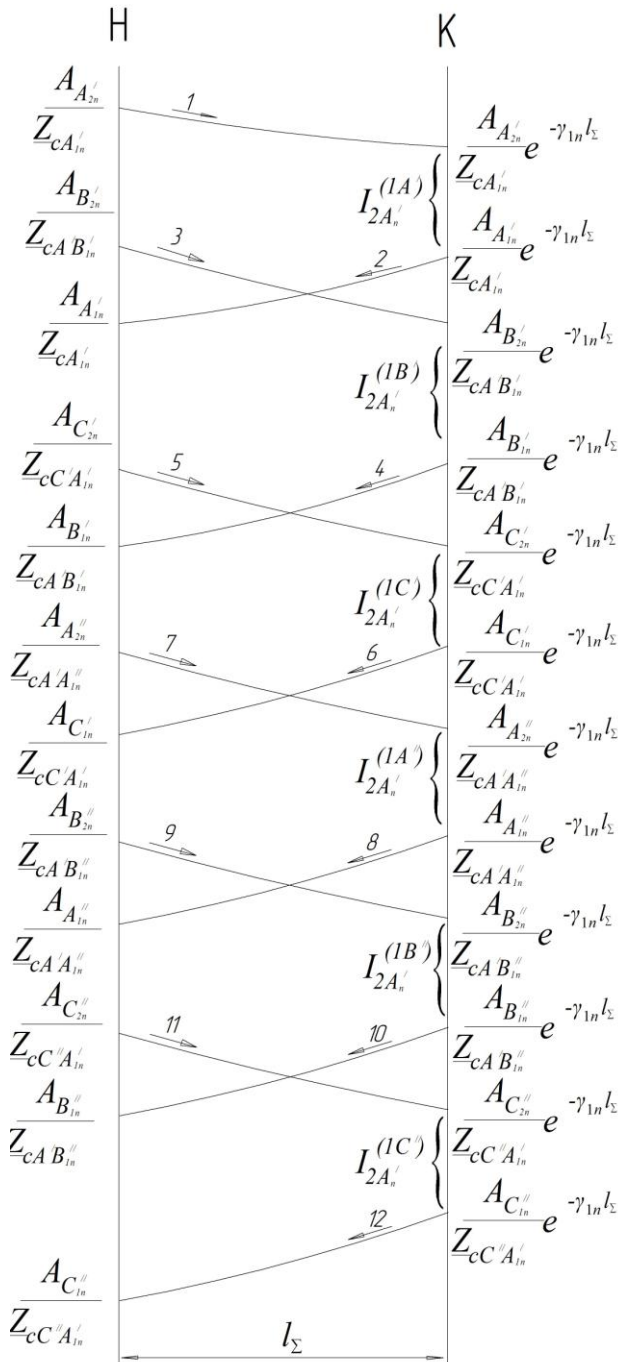
Fig. 1. The distribution scheme of the electromagnetic field wave amplitudes along the linear wire of a homogeneous section of a double-circuit power transmission line

In the distribution scheme of the electromagnetic field wave amplitudes the currents $i_{2A'}^{(1A')}$, $i_{2A'}^{(1B')}$, $i_{2A'}^{(1C')}$, $i_{2A'}^{(1A'')}$, $i_{2A'}^{(1B'')}$, $i_{2A'}^{(1C'')}$ illustrate part of the electrical energy entering the load.

Integration constants illustrate the amplitudes of the waves of the electromagnetic field. Odd integration constants $A_{A'1} - A_{A'11}$, $A_{B'1} - A_{B'11}$, $A_{C'1} - A_{C'11}$, $A_{A''1} - A_{A''11}$, $A_{B''1} - A_{B''11}$, $A_{C''1} - A_{C''11}$ illustrate the reflected waves, and even integration constants $A_{A'2} - A_{A'12}$, $A_{B'2} - A_{B'12}$, $A_{C'2} - A_{C'12}$, $A_{A''2} - A_{A''12}$, $A_{B''2} - A_{B''12}$, $A_{C''2} - A_{C''12}$ – illustrate the incident waves. The integration constants illustrating the incident waves of electromagnetic field exceed the values of integration constants illustrating the reflected waves.

The implementation of equations (1) and (2) allows us to predict currents and voltages at any point in a double-circuit PTL. For this, one should know reliable information about the primary and secondary parameters of power lines and voltages and currents at the end of the analyzed power lines.

Parameters of the equivalent circuit of power line are usually taken as primary parameters, and self and mutual wave resistances, attenuation coefficients, phase coefficients, etc are usually taken as the secondary parameters. Secondary parameters can be calculated if there is reliable data on the primary parameters. Such information can be taken from the reference literature[2]., but it is not exact.

Gerasimova VG, et al. Production, transmission and distribution of electrical energy. Under total ed. professors of MEI VG , 9nd ed., Sr. Moscow: Publishing House MEI, 2004.

More accurate information can be obtained as a result of appropriate calculations [6 - 13]. But it is a cumbersome procedure, requiring consideration of many factors affecting the amount of the transported electrical energy. The primary parameters of PTL can be determined experimentally [14, 15]. But such an operation also requires implementation of a number of organizational and technical measures. But the obtained information will be reliable. Self-wave impedances illustrate the influence of the intrinsic parameters of a linear wire on the transmission of electrical energy. Mutual wave impedances illustrate the influence of electromagnetic links between the current-conducting parts of power lines on the same process of electrical energy transmission.

Self- and mutual wave impedances are calculated by the formulas:

$$\left.\begin{array}{l} \underline{Z}_{cA'i} = \dfrac{\Delta}{\gamma_i \Delta_{1A'}} \; ; \; \underline{Z}_{cA'B'i} = \dfrac{\Delta}{\gamma_i \Delta_{2A'}} \; ; \; \underline{Z}_{cC A'i} = \dfrac{\Delta}{\gamma_i \Delta_{3A'}} \; ; \\[3mm] \underline{Z}_{cA'A''i} = \dfrac{\Delta}{\gamma_i \Delta_{4A'}} \; ; \; \underline{Z}_{cA'B''i} = \dfrac{\Delta}{\gamma_i \Delta_{5A'}} \; ; \; \underline{Z}_{cC''A'i} = \dfrac{\Delta}{\gamma_i \Delta_{6A'}} \; , \end{array}\right\} \qquad (3)$$

where Δ, $\Delta_{1A'}$, $\Delta_{2A'}$, $\Delta_{3A'}$, $\Delta_{4A'}$, $\Delta_{5A'}$, $\Delta_{6A'}$ are the determinants of the system of equations that make up the mathematical model of distribution of electrical energy over a homogeneous section of a double-circuit power line [4].

The following equality confirms that the current at the end of the linear wire A' is derived from six pairs of waves of the electromagnetic field:

$$\dot{I}_{2A'} = \frac{1}{6}\left(\dot{I}_{2A'}^{(1)} + \dot{I}_{2A'}^{(2)} + \dot{I}_{2A'}^{(3)} + \dot{I}_{2A'}^{(4)} + \dot{I}_{2A'}^{(5)} + \dot{I}_{2A'}^{(6)} \right), \qquad (4)$$

where $\dot{I}_{2A'}^{(1)}$, $\dot{I}_{2A'}^{(2)}$, $\dot{I}_{2A'}^{(3)}$, $\dot{I}_{2A'}^{(4)}$, $\dot{I}_{2A'}^{(5)}$ and $\dot{I}_{2A'}^{(6)}$ are the current shares at the end of a linear wire A' from each pair of waves of electromagnetic field. The equation for the first current component in expression (4) is the following:

$$\dot{I}_{2A'}^{(1)} = \dot{I}_{2A'}^{(1A')} + \dot{I}_{2A'}^{(1B')} + \dot{I}_{2A'}^{(1C')} + \dot{I}_{2A'}^{(1A'')} + \dot{I}_{2A'}^{(1B'')} + \dot{I}_{2A'}^{(1C'')} \; , \qquad (5)$$

where $\dot{I}_{2A'}^{(1A')}$ is the self-current at the end of the linear wire A' from the first pair of waves of electromagnetic field; $\dot{I}_{2A'}^{(1B')}$, $\dot{I}_{2A'}^{(1C')}$, $\dot{I}_{2A'}^{(1A'')}$, $\dot{I}_{2A'}^{(1B'')}$, $\dot{I}_{2A'}^{(1C'')}$ are the currents at the end of the linear wire A', induced from neighboring wires from the first pair of electromagnetic field waves. For the remaining components of current in expression (4), the current shares will be found in a similar way.

By substituting equation (5) into equation (4) we obtain:

$$\begin{aligned} \dot{I}_{2A'} = \frac{1}{6}\Big(& \dot{I}_{2A'}^{(1A')} + \dot{I}_{2A'}^{(1B')} + \dot{I}_{2A'}^{(1C')} + \dot{I}_{2A'}^{(1A'')} + \dot{I}_{2A'}^{(1B'')} + \dot{I}_{2A'}^{(1C'')} + \\ & + \dot{I}_{2A'}^{(2A')} + \dot{I}_{2A'}^{(2B')} + \dot{I}_{2A'}^{(2C')} + \dot{I}_{2A'}^{(2A'')} + \dot{I}_{2A'}^{(2B'')} + \dot{I}_{2A'}^{(2C'')} + \\ & + \dot{I}_{2A'}^{(3A')} + \dot{I}_{2A'}^{(3B')} + \dot{I}_{2A'}^{(3C')} + \dot{I}_{2A'}^{(3A'')} + \dot{I}_{2A'}^{(3B'')} + \dot{I}_{2A'}^{(3C'')} + \\ & + \dot{I}_{2A'}^{(4A')} + \dot{I}_{2A'}^{(4B')} + \dot{I}_{2A'}^{(4C')} + \dot{I}_{2A'}^{(4A'')} + \dot{I}_{2A'}^{(4B'')} + \dot{I}_{2A'}^{(4C'')} + \\ & + \dot{I}_{2A'}^{(5A')} + \dot{I}_{2A'}^{(5B')} + \dot{I}_{2A'}^{(5C')} + \dot{I}_{2A'}^{(5A'')} + \dot{I}_{2A'}^{(5B'')} + \dot{I}_{2A'}^{(5C'')} + \\ & + \dot{I}_{2A'}^{(6A')} + \dot{I}_{2A'}^{(6B')} + \dot{I}_{2A'}^{(6C')} + \dot{I}_{2A'}^{(6A'')} + \dot{I}_{2A'}^{(6B'')} + \dot{I}_{2A'}^{(6C'')} \Big), \end{aligned} \qquad (6)$$

The distribution scheme of the electromagnetic field wave amplitudes (Fig. 1) shows that for the end of the linear wire A' the following relation will be valid, which is found as the ratio of the integration constants to the wave impedance:

$$\dot{I}_{2A'}^{(1A')} = \frac{A_{A'2}}{\underline{Z}_{cA'1}} e^{-\gamma_i l_\Sigma} - \frac{A_{A'1}}{\underline{Z}_{cA'1}} e^{\gamma_i l_\Sigma} \; ; \qquad (7)$$

To simplify the subsequent presentation, we perform a replacement using the symbol B:

$$B_{A'1} = A_{A'1}e^{\gamma_1 l_\Sigma} \; ; \; B_{A'2} = A_{A'2}e^{-\gamma_1 l_\Sigma} \, , \tag{8}$$

The remaining integration constants are reduced to equations of the form (8).

By substitution of equation (8) into expression (7), we obtain:

$$\dot{i}_{2A'}^{(1A')} = \frac{B_{A'2}}{\underline{Z}_{cA'1}} - \frac{B_{A'1}}{\underline{Z}_{cA'1}} \, .$$

Thus, the integration constants from the end of a double-circuit power transmission line are defined as the sum of the product of the current and wave impedance and the integration constant:

$$B_{A'2} = \dot{i}_{2A'}^{(1A')}\underline{Z}_{cA'1} + B_{A'1} \, .$$

This equality establishes a quantitative relationship between the integration constants at the beginning and at the end of a double-circuit power transmission line. To calculate the integration constants, it is necessary to know the numerical values of almost all components of the output current. To find them, one can use the fact that each component of current is inversely proportional to the corresponding wave resistance. In this case, taking into account formulas (3), the following equality is true:

$$\left. \begin{array}{l} \dfrac{\dot{i}_{2A'}^{(1A')}}{\dot{i}_{2A'}^{(2A')}} = \dfrac{\underline{Z}_{cA'2}}{\underline{Z}_{cA'1}} = \dfrac{\gamma_1}{\gamma_2} \; ; \; \dfrac{\dot{i}_{2A'}^{(1A')}}{\dot{i}_{2A'}^{(3A')}} = \dfrac{\underline{Z}_{cA'3}}{\underline{Z}_{cA'1}} = \dfrac{\gamma_1}{\gamma_3} \; ; \; \dfrac{\dot{i}_{2A'}^{(1A')}}{\dot{i}_{2A'}^{(4A')}} = \dfrac{\underline{Z}_{cA'4}}{\underline{Z}_{cA'1}} = \dfrac{\gamma_1}{\gamma_4} \; ; \\[2em] \dfrac{\dot{i}_{2A'}^{(1A')}}{\dot{i}_{2A'}^{(5A')}} = \dfrac{\underline{Z}_{cA'5}}{\underline{Z}_{cA'1}} = \dfrac{\gamma_1}{\gamma_5} \; ; \; \dfrac{\dot{i}_{2A'}^{(1A')}}{\dot{i}_{2A'}^{(6A')}} = \dfrac{\underline{Z}_{cA'6}}{\underline{Z}_{cA'1}} = \dfrac{\gamma_1}{\gamma_6} \, . \end{array} \right\} \tag{9}$$

The derived equations (9) allow us to express all currents through the current $\dot{i}_{2A}^{(1A')}$:

$$\dot{i}_{2A'}^{(2A')} = \frac{\gamma_2}{\gamma_1}\dot{i}_{2A'}^{(1A')}; \; \dot{i}_{2A'}^{(3A')} = \frac{\gamma_3}{\gamma_1}\dot{i}_{2A'}^{(1A')}; \; \dot{i}_{2A'}^{(4A')} = \frac{\gamma_4}{\gamma_1}\dot{i}_{2A'}^{(1A')}; \; \dot{i}_{2A'}^{(5A')} = \frac{\gamma_5}{\gamma_1}\dot{i}_{2A'}^{(1A')}; \; \dot{i}_{2A'}^{(6A')} = \frac{\gamma_6}{\gamma_1}\dot{i}_{2A'}^{(1A')}. \tag{10}$$

By substitution expression (10) into equations (6), we obtain an equation in one unknown. In this case unknown is the current $\dot{i}_{2A'}^{(1A')}$. Solution of this equation makes it possible to determine the formula for calculating this current:

$$\dot{i}_{2A'}^{(1A')} = \frac{6\dot{i}_{2A'}\gamma_1\Delta_{1A'} + 0}{\left(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6\right)\left(\Delta_{1A'} + \Delta_{2A'} + \Delta_{3A'} + \Delta_{4A'} + \Delta_{5A'} + \Delta_{6A'}\right)} \, . \tag{11}$$

Voltage at the end of the line wire A', taking into account equalities (8), can be rewritten as follows:

$$6\dot{U}_{1A'} = 2B_{A'1} + 2B_{A'3} + 2B_{A'5} + 2B_{A'7} + 2B_{A'9} + 2B_{A'11} +$$

$$+ \frac{6\dot{i}_{2A'}\Delta_{1A'}\left(\gamma_1\underline{Z}_{cA'1} + \gamma_2\underline{Z}_{cA'2} + \gamma_3\underline{Z}_{cA'3} + \gamma_4\underline{Z}_{cA'4} + \gamma_5\underline{Z}_{cA'5} + \gamma_6\underline{Z}_{cA'6}\right)}{\left(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6\right)\left(\Delta_{1A'} + \Delta_{2A'} + \Delta_{3A'} + \Delta_{4A'} + \Delta_{5A'} + \Delta_{6A'}\right)} =$$

$$= 2B_{A'1} + 2B_{A'3} + 2B_{A'5} + 2B_{A'7} + 2B_{A'9} + 2B_{A'11} + a \, . \tag{12}$$

The initial task was to obtain integration constants, which make it possible to determine voltage and current anywhere in the double-circuit power transmission line. There are six of them in equation (12). To calculate them, one needs another 5 (five) equations. They can be obtained using the derivatives of equation (1) with respect to variable *l*.

The second derivative of equation (1) with respect to variable *l* for the end of the linear wire A' taking into account expression (12) takes the form:

$$\frac{d^2\dot{U}_{2A'}}{dl^2} = \frac{1}{6}\Big[2\gamma_1^2 B_{A'1} + 2\gamma_2^2 B_{A'3} + 2\gamma_3^2 B_{A'5} + 2\gamma_{4n}^2 B_{A'7} + 2\gamma_5^2 B_{A'9} +$$

$$+ 2\gamma_6^2 B_{A'11} + \frac{6\dot{i}_{2A'}\Delta_{1A'}\left(\gamma_1^3\underline{Z}_{cA'1} + \gamma_2^3\underline{Z}_{cA'2} + \gamma_3^3\underline{Z}_{cA'3} + \gamma_4^3\underline{Z}_{cA'4} + \right.}{\left(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6\right)\left(\Delta_{1A'} + \Delta_{2A'} + \Delta_{3A'} + \Delta_{4A'} + \right.} \rightarrow$$

$$\rightarrow \frac{+\gamma_5^3 \underline{Z}_{cA'5} + \gamma_6^3 \underline{Z}_{cA'6}}{+\Delta_{5A'} + \Delta_{6A'}}\Bigg)\Bigg] = \frac{1}{6}\Big(2\gamma_1^2 B_{A'1} + 2\gamma_2^2 B_{A'3} + 2\gamma_3^2 B_{A'5} +$$
$$+ 2\gamma_4^2 B_{A'7} + 2\gamma_5^2 B_{A'9} + 2\gamma_6^2 B_{A'11} + b\Big). \tag{13}$$

When analyzing the results of transmission of electrical energy through power lines, it is necessary to solve differential equations of the second order. This solution is associated with the search for numerical values of the propagation constants. The second derivative of the phase voltage in a linear wire A' with respect to variable l can be determined through the primary parameters of a double-circuit power transmission line [4, 5]:

$$\frac{d^2\dot{U}_{2A'}}{dl^2} = \Big[\underline{Z}_{0A'}\big(\underline{Y}_{0A'0} + \underline{Y}_{0A'B'} + \underline{Y}_{0A'A''} + \underline{Y}_{0A'B''} + \underline{Y}_{0C'A'} +$$
$$+ \underline{Y}_{0C''A'}\big) - \underline{Y}_{0A'B'}\underline{Z}_{0A'B'} - \underline{Y}_{0A'A''}\underline{Z}_{0A'A''} - \underline{Y}_{0C'A'}\underline{Z}_{0C'A'} - \underline{Y}_{0A'B''}Z_{0A'B''} -$$
$$- \underline{Y}_{0C''A'}\underline{Z}_{0C''A'n}\Big]\dot{U}_{2A'} + \Big[\underline{Z}_{0A'B'}\big(\underline{Y}_{0B'0} + \underline{Y}_{0B'C'} + \underline{Y}_{0B'B''} + \underline{Y}_{0B'C''} +$$
$$+ \underline{Y}_{0B'A''} + \underline{Y}_{0A'B'}\big) - \underline{Y}_{0A'B'}\underline{Z}_{0A'} - \underline{Y}_{0B'C'}\underline{Z}_{0C'A'} - \underline{Y}_{0B'A''}\underline{Z}_{0A'A''} -$$
$$- \underline{Y}_{0B'B''}\underline{Z}_{0A'B''} - \underline{Y}_{0B'C''}\underline{Z}_{0C''A'}\Big]\dot{U}_{2B'} + \Big[\underline{Z}_{0C'A'}\big(\underline{Y}_{0C'0} + \underline{Y}_{0C'A''} +$$
$$+ \underline{Y}_{0C'A'} + \underline{Y}_{0C'C''} + \underline{Y}_{0B'C'} + \underline{Y}_{0B''C'}\big) - \underline{Y}_{0C'A'}\underline{Z}_{0A'} - \underline{Y}_{0B'C'}\underline{Z}_{0A'B'} -$$
$$- \underline{Y}_{0C'A''}\underline{Z}_{0A'A''} - \underline{Y}_{0B''C'}\underline{Z}_{0A'B''} - \underline{Y}_{0C'C''}\underline{Z}_{0C''A'}\Big]\dot{U}_{2C'} +$$
$$+ \Big[\underline{Z}_{0A'A''}\big(\underline{Y}_{0A''0} + \underline{Y}_{0A''B''} + \underline{Y}_{0A''A''} + \underline{Y}_{0C''A''} + \underline{Y}_{0B'A''} + \underline{Y}_{0C'A''}\big) -$$
$$- \underline{Y}_{0A''B''}\underline{Z}_{0A'B''} - \underline{Y}_{0C''A''n}\underline{Z}_{0C''A'} - \underline{Y}_{0A'A''}\underline{Z}_{0A'} - \underline{Y}_{0B'A''}\underline{Z}_{0A'B'} -$$
$$- \underline{Y}_{0C'A''}\underline{Z}_{0C'A'}\Big]\dot{U}_{2A''} + \Big[\underline{Z}_{0A'B''}\big(\underline{Y}_{0B''C''} + \underline{Y}_{0B''0} + \underline{Y}_{0B''C'} +$$
$$+ \underline{Y}_{0A'B''} + \underline{Y}_{0B'B''} + \underline{Y}_{0A''B''}\big) - \underline{Y}_{0B''C''}\underline{Z}_{0C''A'} - \underline{Y}_{0A'B''}\underline{Z}_{0A'} -$$
$$- \underline{Y}_{0B'B''}\underline{Z}_{0A'B'} - \underline{Y}_{0B''C'}\underline{Z}_{0C'A'} - \underline{Y}_{0A''B''}\underline{Z}_{0A'A''}\Big]\dot{U}_{2B''} +$$
$$+ \Big[\underline{Z}_{0C''A'}\big(\underline{Y}_{0C''0} + \underline{Y}_{0C''A'} + Y_{0C''A''} + Y_{0B'C''} + Y_{0C'C''} +$$
$$+ Y_{0B''C''}\big) - Y_{0C''A'}Z_{0A'} - Y_{0B'C''}Z_{0A'B'} - \underline{Y}_{0C'C''}\underline{Z}_{0C'A'} -$$
$$- \underline{Y}_{0C''A''}\underline{Z}_{0A'A''} - \underline{Y}_{0B''C''}\underline{Z}_{0A'B''}\Big]\dot{U}_{2C''} =$$
$$= b_1\dot{U}_{2A'} + b_2\dot{U}_{2B'} + b_3\dot{U}_{2C'} + b_4\dot{U}_{2A''} + b_5\dot{U}_{2B''} + b_6\dot{U}_{2C''}.$$

For the end of the line wire A', this equation can be rewritten as follows:

$$\frac{d^2\dot{U}_{2A'}}{dl^2} = b_1\dot{U}_{2A'} + b_2\dot{U}_{2B'} + b_3\dot{U}_{2C'} + b_4\dot{U}_{2A''} + b_5\dot{U}_{2B''} +$$
$$+ b_6\dot{U}_{2C''} = b_0. \tag{14}$$

When combining equation (14) with equation (13), the following expression is obtained:

$$6b_0 - b = 2\gamma_1^2 B_{A'1} + 2\gamma_2^2 B_{A'3} + 2\gamma_3^2 B_{A'5} + 2\gamma_4^2 B_{A'7} +$$
$$+ 2\gamma_5^2 B_{A'9} + 2\gamma_6^2 B_{A'11}. \tag{15}$$

The fourth derivative of equation (1) with respect to variable l for the beginning of a linear wire taking into account equalities (8) takes the form:

$$\frac{d^4\dot{U}_{2A'}}{dl^4} = \frac{1}{6}\Big(2\gamma_1^4 B_{A'1} + 2\gamma_{2n}^4 B_{A'3} + 2\gamma_3^4 B_{A'5} + 2\gamma_4^4 B_{A'7} +$$
$$+ 2\gamma_5^4 B_{A'9} + 2\gamma_6^4 B_{A'11} + c\Big). \tag{16}$$

The fourth derivative of voltage at the end of the linear wire with respect to variable l can be represented as follows:

$$\frac{d^4\dot{U}_{2A'}}{dl^4} = c_1\dot{U}_{2A'} + c_2\dot{U}_{2B'} + c_3\dot{U}_{2C'} + c_4\dot{U}_{2A''} + c_5\dot{U}_{2B''} + c_6\dot{U}_{2C''} = c_0. \tag{17}$$

By combining expression (17) with equation (16) we obtain:

$$6c_0 - c = 2\gamma_1^4 B_{A'1} + 2\gamma_2^4 B_{A'3} + 2\gamma_3^4 B_{A'5} + 2\gamma_4^4 B_{A'7} + 2\gamma_5^4 B_{A'9} + 2\gamma_6^4 B_{A'11}. \tag{18}$$

The sixth derivative of equation (1) with respect to variable l for the beginning of a linear wire, taking into account equalities (8), will take the form:

$$\frac{d^6 \dot{U}_{2A'}}{dl^6} = \frac{1}{6}\left(2\gamma_1^6 B_{A'1} + 2\gamma_2^6 B_{A'3} + 2\gamma_3^6 B_{A'5} + 2\gamma_4^6 B_{A'7} + 2\gamma_5^6 B_{A'9} + 2\gamma_6^6 B_{A'11} + d\right). \tag{19}$$

The sixth derivative of voltage at the end of the linear wire A' with respect to variable l can be also written as follows:

$$\frac{d^6 \dot{U}_{2A'}}{dl^6} = d_1 \dot{U}_{2A'} + d_2 \dot{U}_{2B'} + d_3 \dot{U}_{2C'} + d_4 \dot{U}_{2A''} + d_5 \dot{U}_{2B''} + d_6 \dot{U}_{2C''} = d_0. \tag{20}$$

The combination of equation (20) with equation (19) gives the following expression:

$$6d_0 - d = 2\gamma_1^6 B_{A'1} + 2\gamma_2^6 B_{A'3} + 2\gamma_3^6 B_{A'5} + 2\gamma_4^6 B_{A'7} + 2\gamma_5^6 B_{A'9} + 2\gamma_6^6 B_{A'11}. \tag{21}$$

The eighth derivative of equation (1) with respect to variable l for the beginning of a linear wire A', taking into account equalities (8), takes the form:

$$\frac{d^8 \dot{U}_{2A'}}{dl^8} = \frac{1}{6}\left(2\gamma_1^8 B_{A'1} + 2\gamma_2^8 B_{A'3} + 2\gamma_3^8 B_{A'5} + 2\gamma_4^8 B_{A'7} + 2\gamma_5^8 A B_{A'9} + 2\gamma_6^8 B_{A'11} + f\right). \tag{22}$$

The eighth derivative of voltage at the end of the linear wire A' with respect to variable l can be written as follows:

$$\frac{d^8 \dot{U}_{2A'}}{dl^8} = f_1 \dot{U}_{2A'} + f_2 \dot{U}_{2B'} + f_3 \dot{U}_{2C'} + f_4 \dot{U}_{2A''} + f_5 \dot{U}_{2B''} + f_6 \dot{U}_{2C''} = f_0. \tag{23}$$

Equation (23) together with equation (22) form the following expression:

$$6f_0 - f = 2\gamma_1^8 B_{A'1} + 2\gamma_2^8 B_{A'3} + 2\gamma_3^8 B_{A'5} + 2\gamma_4^8 B_{A'7} + 2\gamma_5^8 B_{A'9} + 2\gamma_6^8 B_{A'11}. \tag{24}$$

The tenth derivative of equation (1) with respect to variable l for the beginning of a linear wire A', taking into account equalities (8), will take the following form:

$$\frac{d^{10} \dot{U}_{2A'}}{dl^{10}} = \frac{1}{6}\left(2\gamma_1^{10} B_{A'1} + 2\gamma_2^{10} B_{A'3} + 2\gamma_3^{10} B_{A'5} + 2\gamma_4^{10} B_{A'7} + 2\gamma_5^{10} B_{A'9} + 2\gamma_6^{10} B_{A'11} + h\right). \tag{25}$$

So, the tenth derivative of voltage at the end of the linear wire A' with respect to variable l takes the form:

$$\frac{d^{10} \dot{U}_{2A'}}{dl^{10}} = h_1 \dot{U}_{2A'} + h_2 \dot{U}_{2B'} + h_3 \dot{U}_{2C'} + h_4 \dot{U}_{2A''} + h_5 \dot{U}_{2B''} + h_6 \dot{U}_{2C''} = h_0. \tag{26}$$

By combining equation (26) with equation (25) we get the following:

$$6h_0 - h = 2\gamma_1^{10} B_{A'1} + 2\gamma_2^{10} B_{A'3} + 2\gamma_3^{10} B_{A'5} + 2\gamma_4^{10} B_{A'7} + 2\gamma_5^{10} B_{A'9} + 2\gamma_6^{10} B_{A'11}. \tag{27}$$

The joint solution of equations (12), (15), (18), (21), (24) and (27) will allow us to form equalities for calculating the odd integration constants from $A_{A'1}$ to $A_{A'11}$. The even constants of integration from $A_{A'2}$ to $A_{A'12}$ are calculated by formulas (8).

In a similar way, the remaining integration constants for each linear wire of a double-circuit power transmission line are determined. This can be carried out under the condition of availability of reliable information about the numerical values of the primary parameters of a homogeneous section of a three-phase power transmission line of double-circuit design and output voltages and currents at the frequency of each harmonic component.

Results

A method has been developed for determining the numerical values of integration constants at known output phase voltages and linear currents. The integration constants calculated in this way will provide insight into the propagation of electromagnetic field waves in the linear wires of power lines. In addition, the information obtained allows predicting the results of transmission of electrical energy through a double-circuit power transmission line.

Conclusion

The result of this study can be applied in engineering practice at the stage of design, reconstruction and operation of double-circuit power transmission lines. Calculation of the integration constants, currents and voltages, and the operational parameters as a whole, will make it possible to predict the result of electrical energy transmission along the homogeneous sections of a double-circuit power transmission line.

With a minor modification, the proposed methodology for calculating the integration constants, and therefore the design of electrical energy transmission results, can be extended to heterogeneous sections of power lines, and to the entire transmission line as a whole. It can serve as an example of the development of similar techniques for power lines of other versions.

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