



## SUPERSONIC FLOW OF TWO-PHASE GAS-DROPLET FLOWS IN NOZZLES

A.I. Sharapov, A.A. Chernykh, A.V. Peshkova

Lipetsk State Technical University, Lipetsk, Russia

**Abstract:** For practical applications, the description of processes occurring during the flow of two-phase gas-liquid mixtures requires a simple physical and mathematical model that describes the behavior of a two-phase medium in the entire range of phase concentrations changes and in a wide range of pressure changes. Problems of this kind arise in various branches of industry and technology. In the space industry, one often has to deal with the movement of various gases in rocket nozzles, consider the combustion, condensation of various vapors on the nozzle walls and their further impact on the velocity sublayer at the nozzle wall. The large acoustic effect arising from the engines affects the gas-liquid mixture in the nozzles of rocket engines. In the metal industry, metal cooling occurs with the help of nozzles in which the emulsion mixture is supplied under high overpressure. But this is only a short list of applied issues in which one has to deal with a problem of this type. The paper presents the results and directions of study of the problems of two-phase dispersed gas-droplet flows in the nozzles. The main methods of investigation of two-phase heterogeneous flows are described. The main characteristics of heterogeneous two-phase flows in the nozzles, which were confirmed by experimental results, are presented. The calculation of the air-droplet flow in the Laval nozzle is given. The technique, which is based on integral energy equations for two-phase dispersed flows, is described. The main problems and questions concerning the further description and studying of two-component flows are stated.

**Keywords:** gas-droplet flows, heterogeneous medium, sound propagation, sound velocity, Laval nozzle, quasi-equilibrium flow, critical parameters, medium compressibility, Witoszynskyj nozzle, acoustics of dispersed medium.

**For citation:** Sharapov AI, Chernykh AA, Peshkova AV. Supersonic flow of two-phase gas-droplet flows in nozzles. *Power engineering: research, equipment, technology*.2019; 21(3):86-98. (In Russ). doi:10.30724/1998-9903-2019-21-3-86-98.

## СВЕРХЗВУКОВОЕ ТЕЧЕНИЕ ДВУХФАЗНЫХ ГАЗОКАПЕЛЬНЫХ ПОТОКОВ В СОПЛАХ

А.И. Шарапов, А.А. Черных, А.В. Пешкова

Липецкий государственный технический университет, г. Липецк, Россия

sharapov-lipetsk@yandex.ru, Peshkova\_Nastja@mail.ru

**Резюме:** Для практических приложений описания процессов, происходящих при течении двухфазных газожидкостных смесей, необходима простая физико-математическая модель, описывающая поведение двухфазной среды во всем диапазоне изменения концентраций фаз и в широком диапазоне изменения давлений. Проблемы такого рода встают в разных отраслях промышленности и техники. В космической промышленности часто приходится сталкиваться с движением различных газов в соплах ракет, рассматривать сгорание, конденсацию различных паров на стенках сопел и дальнейшее их

*влияние на скоростной подслои у стенки сопла. Большой акустический эффект, исходящий от двигателей, влияет на газожидкостную смесь в соплах ракетных двигателей. В металлопромышленности имеет место охлаждение металла с помощью форсунок, в которых эмульсионная смесь подаётся под высоким избыточным давлением. Но это лишь краткий перечень прикладных задач, в которых приходится сталкиваться с проблемой такого типа. В работе приведены результаты и направления исследования проблематики течения двухфазных дисперсных газокapельных потоков в соплах за последнее время. Изложены основные методы исследования двухфазных гетерогенных потоков. Приводятся основные характеристики протекания гетерогенных двухфазных потоков в соплах, которые были подтверждены опытными результатами. Приводится расчёт течения воздушно-капельного потока в сопле Лавалья. Изложена методика, которая опирается на интегральные энергетические уравнения для двухфазных дисперсных потоков. Изложены основные проблемы и вопросы, касающиеся дальнейшего описания и изучения двухкомпонентных потоков. В расчетах пренебрегается структура двухфазного потока и рассматривают его течение как односкоростной однотемпературный континуум.*

**Ключевые слова:** *газокapельные потоки, гетерогенная среда, распространение звука, скорость звука, сопло Лавалья, квазиравновесное течение, критические параметры, сжимаемость среды, сопло Витошинского, акустика дисперсных сред*

### Introduction

To describe the processes occurring during the two-phase gas-liquid mixtures flow, a simple physical and mathematical model is needed that shows the behavior of a two-phase medium in the entire range of changes in phase concentrations and in a wide range of pressure changes. Problems of this kind arise in various branches of industry and technology. In the space industry, one often has to deal with the movement of various gases in rocket nozzles, consider their combustion, condensation of various vapors on the nozzle walls and their further impact on the velocity sublayer near the nozzle wall. The large acoustic effect arising from the engines affects the gas-liquid mixture in the nozzles of rocket engines. In metal industry, metal cooling is made using nozzles in which the emulsion mixture is supplied under high overpressure.

The study of motion of various two-phase media is currently of great interest for the aircraft industry, engineering, and for the design of water circuit of nuclear reactors. Due to the complexity of the processes of interphase interaction, the general approaches for these types of two-phase mixtures are not formalized due to the peculiarities of their behavior. This issue was already addressed in the 30-40s of the last century, but issues related to determination the speed of sound [1-4], description the flow in the presence of phase transitions, etc., are still not systematized and have not been properly studied. This interest is mainly caused by finding universal relations [2] for describing various flow parameters in dispersed multicomponent media, since motion of such flows, for example, in nozzles is associated with various heat transfer processes, as well as processes accompanied by phase transitions. This can be attributed to various gas-droplet media in which the dispersed phase in the form of liquid droplets is in suspension.

It is also worth noting that a small number of works [2, 4] are devoted to the study of gas-droplet systems at sufficiently high mass contents of the droplet phase ( $\varphi \geq 5\%$ ).

The study of propagation of sound waves in various media is inextricably linked with heat and mass transfer processes, but the authors [1, 5, 6] state that heat transfer processes are not involved in the propagation of acoustic waves in stationary two-phase media. As for the unjustified increase in speed at  $\varphi < 0.5$ , the situation has an inverse physical formulation. The gaseous medium itself begins to prevail over the liquid one, and the gaseous medium, which begins to fill the entire volume, becomes a source of waves itself. As a result of long-range order destruction by

perturbations, a state with slowly decreasing waves arises, therefore, the energy that the surface layer of the  $\delta$ -droplet begins to emit is amplified, and two wave fronts are obtained.

As was noted above, an interpretation based on linear theory can no longer be applied [1, 5], since the so-called dispersion phenomena are observed during the propagation of sound waves: reflection, absorption, etc. If one considers the “liquid – gas” interface, an abrupt change in the speed of sound will be observed at its boundary [5-10].

We address a two-phase medium of the “droplets–gas” type in which a wave propagates with a length comparable to the free-path length between the droplets. The concept of the speed of sound should be taken into account. It is appropriate to use the methods of molecular kinetic theory, in which molecules will be replaced by small droplets (heterogeneous medium). Knudsen's criterion will be applicable to such a system. The description of a system with sufficiently large liquid droplets requires another approach: not from the point of view of compressibility, but with consideration of such a process as a two-component one [1, 6, 10, 11-16]. The system should be divided into separate “droplets–gas” transitions, which together will characterize the entire system. The wave propagation process is similar to the Brownian motion of molecules, as a result of which some assumptions become applicable. Based on these formulas, a comparison of different gas-liquid systems was made.

As a result, methods of probabilistic description of the process or a separate description of the liquid – gas boundary become feasible. There exists the scaling method, when the system is compared with the free path length between individual dispersed inclusions. In this case, for example, a sound wave of a certain frequency flies onto a drop, which generates a series of fronts (also acoustic waves) that begin to propagate from the source in different directions [17, 18]. The generation of waves is associated with a change in the continuum, and therefore in acoustic waves the main change is made by the liquid, which, due to its diffusion motion, is a very good refractive component. Therefore, based on this point of view, one should subdivide the degree of intensity of the wave propagation through such dispersive homogeneous media.

### Research method

Consider the equations for equilibrium and frozen flows. We will proceed from the basic kinetic integral equation for a two-component medium. After simple transformations, it reduces to the equation

$$w = \sqrt{w_0^2 + \frac{2c_p(T_0 - T)}{(1 + \varphi^2 d)}} \quad (1)$$

During a polytropic process, in case when there are heat exchange processes, relation (1) takes the form

$$w = \sqrt{w_0^2 + \frac{2c_p T_0 (1 - \beta^m)}{(1 + \varphi^2 d)}} \quad (2)$$

For an inhibited process, one can obtain the ratio

$$w = \sqrt{w_0^2 + \frac{2c_p T_0 (1 - \beta^m)}{(1 + d)}} \quad (3)$$

Several conclusions can be drawn from these two equations. Firstly, the Mach number for the inhibited flow, with the same pressure and temperature drops, is greater than the Mach number for the equilibrium flow. This is true for any ratio of phase slip coefficients  $\varphi$ .

### Laval nozzle calculation

We give an example of the Laval nozzle calculation for a two-phase vapor-air medium. The pressure change profile is shown in Fig. 1. The initial pressure is 1.2 bar, the output pressure is 0.5 bar.

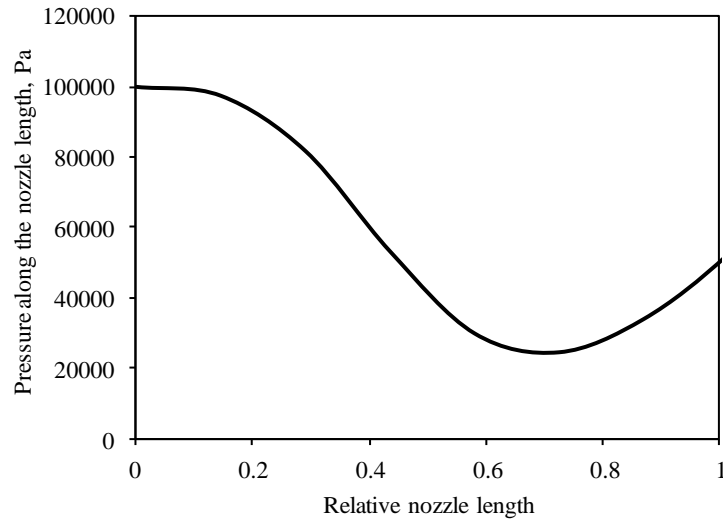


Fig. 1. The profile of pressure changes along the entire length of the Laval nozzle

Consider the change in velocity over the entire nozzle profile for the case of equilibrium air flow. An example of calculation is shown in Fig. 2 for different exponents  $m$ .

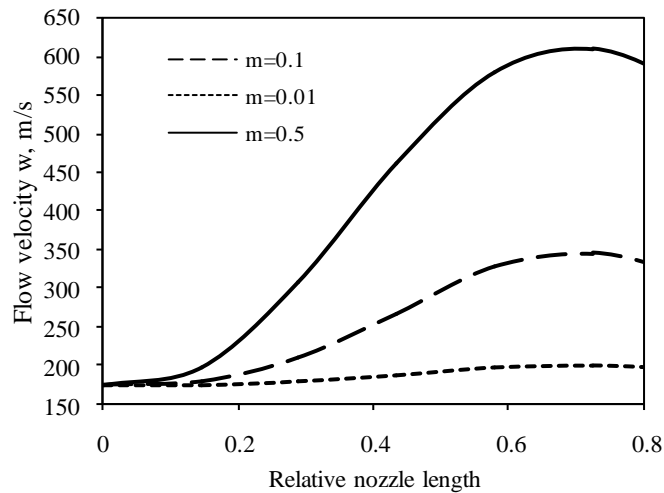


Fig. 2. Change in the air quasi-equilibrium flow rate at different exponents  $m$

It is seen that for the case  $m=0.1$ , the speed of sound is reached in the critical section. For the case  $m=0.5$ , the speed of sound is reached well before the flow reaches the critical section of the nozzle. But such a case is not realized in practice, because the adiabatic index cannot be negative.

Now consider the flow distribution with a small amount of gradually mixed moisture  $\Delta d=0.4$  kg/kg. The initial flow velocity at the entrance to the nozzle is taken equal to 10 m/s. From the graphs in Fig. 3, it can be seen that for  $m=0.5$  the flow rates are different in the critical section, and for the frozen flow this value is greater than for the equilibrium one. The speed of sound is reached later at equilibrium flows in comparison with the frozen flow, as shown in Fig. 3. This figure shows the establishment of the speed of sound for both equilibrium and frozen flows. In this case, a gradual mixing of moisture deforms the velocity profile, and the achievement of the speed of sound for a frozen flow occurs long before the minimum nozzle cross section is reached. As the

exponent  $m$  decreases, the flow continues to expand, passing through the critical section of the Laval nozzle.

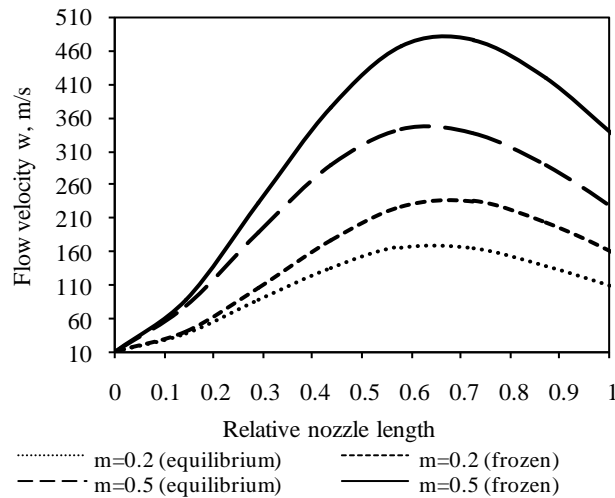


Fig. 3. Change in the gas-droplet flow with a gradually mixed moisture  $\Delta d = 0.4$  kg moist./kg at different exponents  $m$ . For the inhibited flow the speed of the liquid component is two times lower

Figures 4 and 5 show the results of calculating the change in the rates of the frozen and equilibrium two-phase droplet-air flow. These dependences show how the slip coefficient  $\varphi$  affects the flow propagation in the nozzle. An increase in the velocity of the droplet component in the main flow leads to a shift in the maximum of the velocity profile towards the output section, and the case with the opposite effect leads to a shift in the maximum to the input section. Moreover, such an effect is observed only if there is a mixing of small portions of moisture.

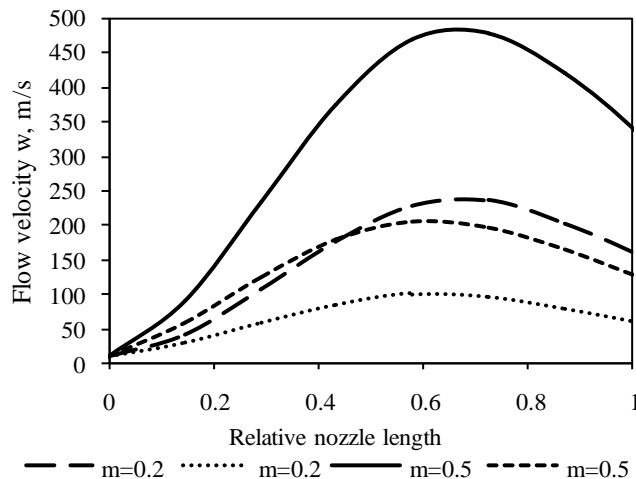


Fig. 4. Change in the gas-droplet inhibited flow with a gradually mixed moisture  $\Delta d = 0.4$  kg moist./kg at different exponents  $m$ . For  $m=0.2$  the speed of the liquid component is two times lower than the gas one, and for  $m=0.5$  the reversed dependence is observed

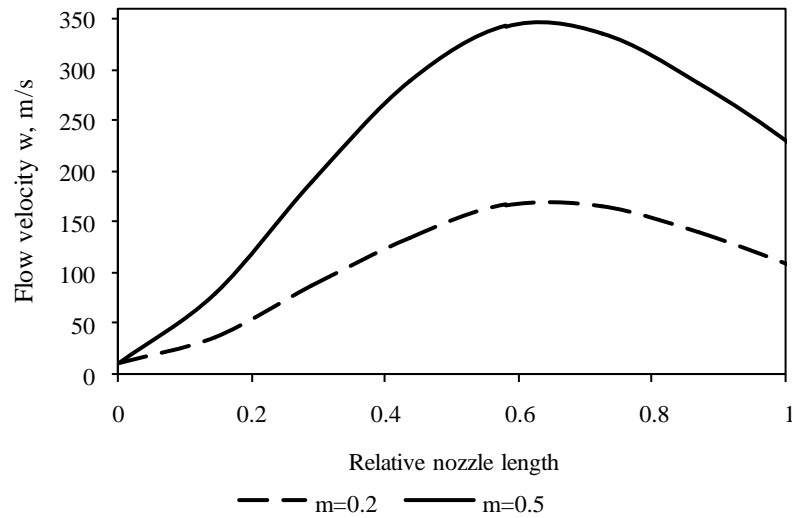


Fig. 5. Change in the gas-droplet quasi-equilibrium flow with a gradually mixed moisture  $\Delta d = 0.4$  kg moist./kg at different exponents  $m$ .

Consider a gas-droplet flow with a moisture content of 1 kg. moist./kg. In fig. 6 it can be seen that for two-phase gas-droplet media the flow velocity does not approach the sonic one in a critical section. Note that the critical velocity for a two-phase flow does not correspond to the speed of sound in gas. The critical velocity becomes equal to the speed of sound only in a gas in which there is no moisture or dispersed inclusions. Since the critical velocity is equal to the speed of sound only for an ideal gas and isoentropic flow, such a property is no longer applicable for a polytropic flow.

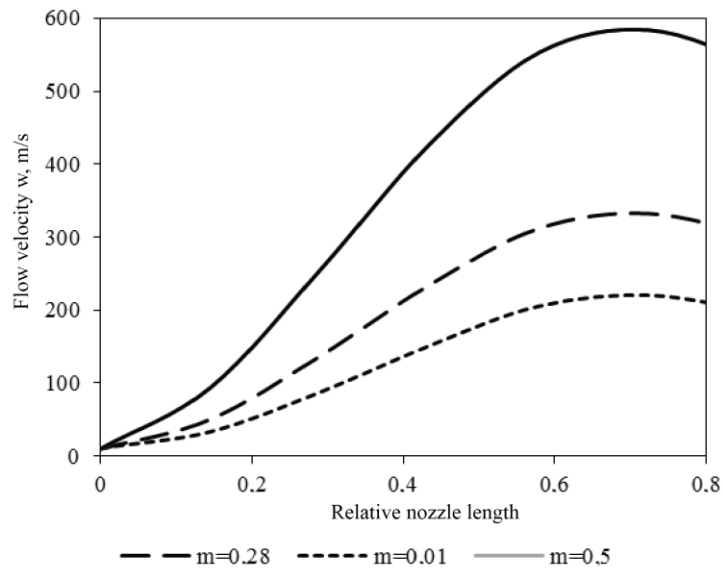


Fig. 6. Change in the gas-droplet equilibrium flow with  $d=1$  kg moist./kg at different exponents  $m$ .

### Taper nozzle calculation

If the area is known, then the speed of each phase can be found, since the gas phase consumption is known,  $G=20$  kg/s, and the phase densities are known. In this case, the droplet phase velocity can be found using the moisture content ratio (3). The remaining graphs were constructed using the iteration method using formula (3). The main ratios that were used in this task are:

$$w_i = \frac{G}{\rho_g F_i} \quad \text{and} \quad w_{i+1} = \frac{G}{\rho_g F_{i+1}} \quad (4)$$

We find the temperature from relation (3), which can be represented by the iteration method as follows:

$$w_{i+1} = \sqrt{w_i^2 + \frac{2c}{p} \frac{(T_{i+1} - T_i)}{(1 + \varphi^2 d)}} \quad (5)$$

The pressure is found from the Mendeleev-Klaiperon equation of gas state in the form

$$\Delta p_i = \rho R_\mu \Delta T_i. \quad (6)$$

Further, one can set the distribution of the nozzle cross-sectional area over its entire dimensionless length, and from here find the flow velocity, since the amount of air  $M$  supplied to the nozzle is known. Then, using formula (6), one can find the temperature difference at each iteration by setting the velocity coefficient. In this case, the velocity coefficient was taken equal to 0.8. It was indicated that in the course of solving the problem the phase pressures were taken equal to each other. In this case, the law of distribution of the cross-sectional area, which was used in solving this problem  $F=f(x)$  is:

$$F = Ax^2 + B,$$

where  $x$  is the dimensionless nozzle length,  $A$  and  $B$  are the constants, which are found from the conditions: for  $F_1 = 0.9 \text{ m}^2$ ,  $x_1 = 0$ , and for  $F_2 = 0.5 \text{ m}^2$ ,  $x_1 = 1$ .

Further the main calculation is presented. The pressure change along the entire dimensionless nozzle length is set by the following laws, which are shown in Figs. 7 and 8.

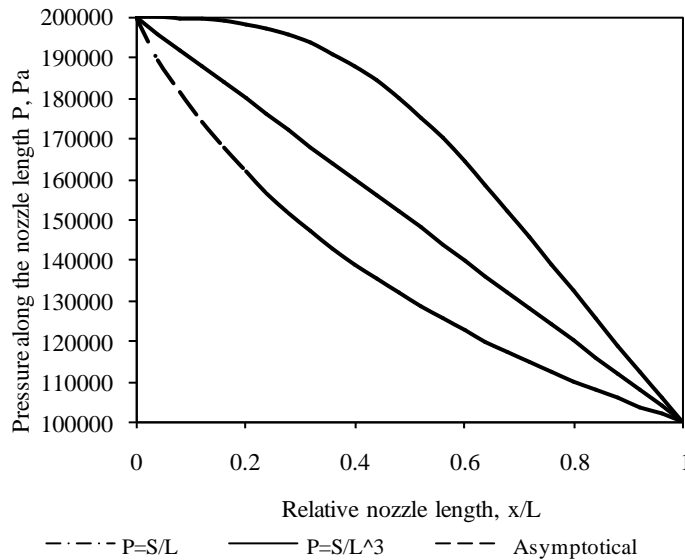


Fig. 7. Pressure change along the entire dimensionless nozzle length

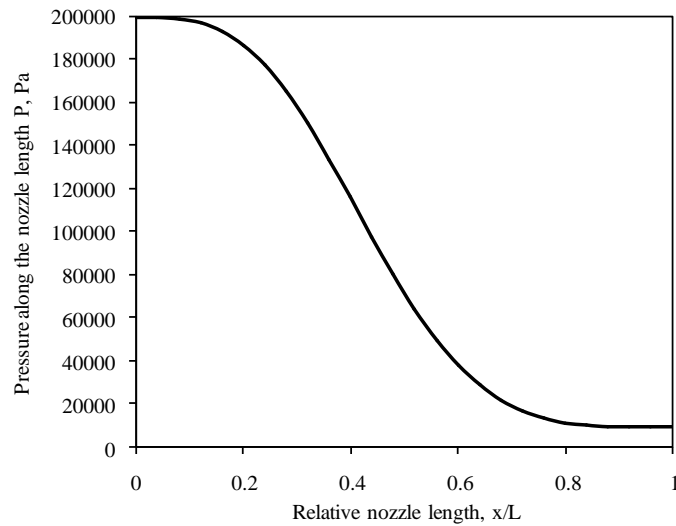


Fig. 8. Pressure change along the entire dimensionless nozzle length according to the exponential law (Witoszynskyj)

Since the temperature in the gas is equal to the temperature in the liquid, the temperature change in the media will be adiabatic:

$$T = T_0 \left( \frac{P}{P_0} \right)^{\frac{k-1}{k}} \quad (7)$$

Temperature distribution in the equilibrium flow is different. Such flows are characterized by polytropic gas compression, therefore, we use the formula:

$$T = T_0 \left( \frac{P}{P_0} \right)^{\frac{n-1}{n}}. \quad (8)$$

For convenience, the polytropic index is taken as 1.2.

Thus, the task of propagation of a two-phase flow of the “liquid droplets-gas” type in nozzles of different geometries was solved and graphs of distribution of the two-phase flow velocity over the dimensionless length of the nozzle were obtained (Figs. 9-11). The solid line in figures for the law, which is shown by the solid line in Fig. 8; small strokes and dash-dotted lines show the approximations for the law depicted in Fig. 7.



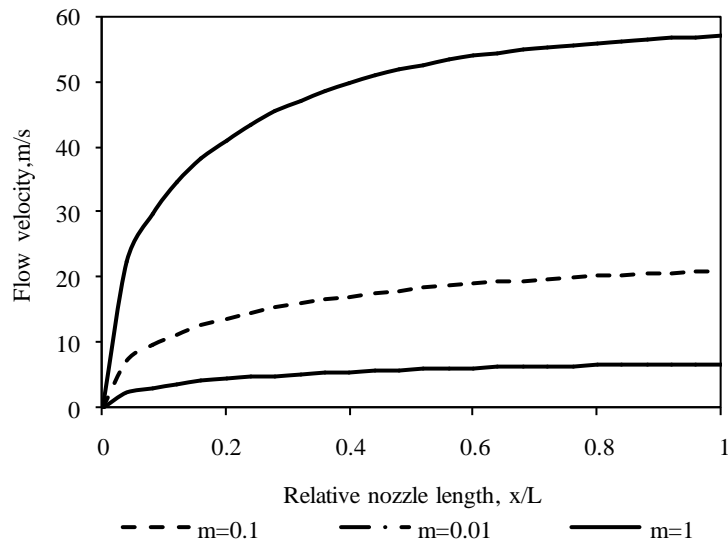


Fig. 9. Relationship between change in the inhibited flow velocity and dimensionless nozzle length at different exponents  $m$  when pressure is distributed according to the hyperbolic law

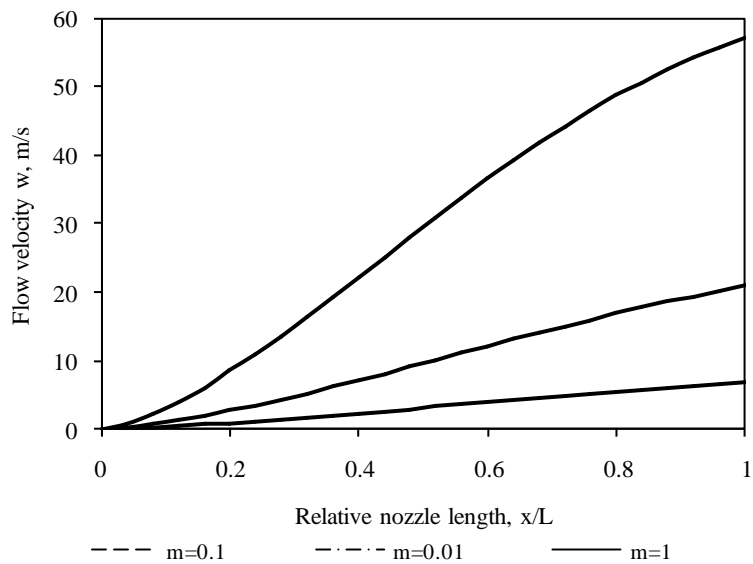


Fig. 9. Relationship between change in the inhibited flow velocity and dimensionless nozzle length at different exponents  $m$  when pressure is distributed according to the cubic law

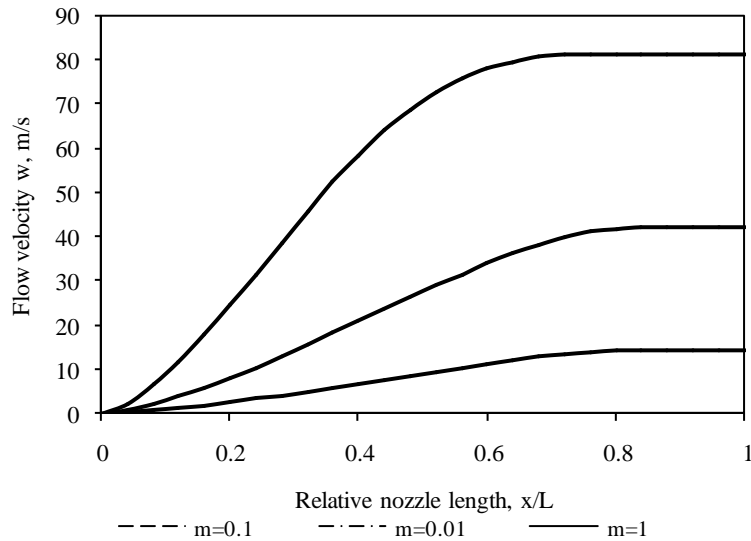


Fig. 10. Relationship between change in the inhibited flow velocity and dimensionless nozzle length at different exponents  $m$  when pressure is distributed according to the exponential law

The dimensionless length is presented as the ratio of the variable value to the maximum one, and the dimensionless diameter in relation to the minimum value. The initial temperature was taken equal to 70 °C, the initial pressure was 200 kPa.

The obtained results show that the most acceptable profile for achieving a high flow velocity is the exponential profile. The same can be said about the achievement of the speed of sound: during profiling the most suitable nozzle, an exponential nozzle is more preferable from the point of view of critical flow parameters. These dependences were obtained for quasi-equilibrium flow. In each case the prevalence of equilibrium or frozen methods of mixture compressibility will be determined by the non-stationary process conditions. Thus, there arises a problem for describing a two-phase system with the same component parts from the point of view of acoustics. And here it is necessary to emphasize that a clear answer on how to describe such a medium in terms of compressibility [1, 10, 11, 14, 16] has not been received yet. Therefore, the concept of sound speed for such a system is no longer applicable. It is worth noting another one property of the Vitoshinsky profile. When  $x/L=0.75$  is reached, the flow velocity remains constant, i.e. critical flow rate was reached at 0.75. The remaining tapering nozzles need to set a large degree of pressure reduction in order to increase the speed of the working jet.

It remains to consider one more question about the applicability of all thermodynamic equations to heterogeneous gas-droplet media, but with rather small droplets. The theoretical and experimental studies for superheated vapor were also carried out, which resulted in formulation of conditions for the applicability of the gas state equation. As a result, for  $M \leq 3$  and a pressure of 1 MPa, it is possible to use the thermodynamic equations of gas state for such gas-droplet flows.

Under certain properties, superheated or saturated vapor can be considered as a homogeneous medium, but the applicability of the basic thermodynamic equations is violated at high supersonic speeds [11, 12]. Therefore, it is worth pointing out that a similar picture of applicability of thermodynamic relations for an ideal gas will be observed for vapor-gas media under the condition that this medium does not contain dispersed inclusions in the form of droplets.

### Conclusions

A two-phase medium is a system that has certain specific features that appear as a result of the propagation of acoustic waves of different frequencies. In this connection, several questions arise that need to be considered: how to characterize such a system from the point of view of sound theory and what is meant by the speed of sound; how to describe the characteristics of such

a system, which includes such critical parameters as  $P_{cr}$  and  $t_{cr}$ ? Therefore, the question about the inapplicability of the concept of the speed of sound to multiphase media is raised, since it is ultimately necessary to consider in each phase its speed of sound. In this case, the analysis of the two-phase flow should be based on the general integral energy equations, taking into account the fact that the gas and condensed phases can have different temperatures, pressure and velocity using a two-fluid flow model; the predominance in each case of equilibrium or frozen methods of compressibility of the mixture will be determined by the conditions of an unsteady process. A two-phase or some other multicomponent system must be described using equation (2); at the same pressure and temperature the Mach number for the inhibited flow is greater than the Mach number for the equilibrium flow. With a high content of the liquid component, the condition of local thermodynamic equilibrium is violated, and one should not use the equations applicable to the gas component.

#### References

1. Chernyh A.A., Peshkova A.V. Nachal'nye svedeniya o teorii rasprostraneniya zvuka v dvuhkomponentnykh smesyakh. 3 Mezhdunarodnaya nauchnaya konferenciya studentov i molodykh uchenykh. «Molodezh' i sistemnaya modernizatsiya strany»; 22-23 may 2018; Kursk, Russia. 2018. pp. 207–211. (In Russ).
2. Nakoryakov V.E., Pokusaev B.G. Schreiber, I.R. Rasprostranenie voln v gazovykh i parozhidkostnykh sredakh. Novosibirsk: Institut teplofiziki SO AN SSSR, 1983. (In Russ).
3. Gubarev V.Ya.. Gazozhidkostnye techeniya v soplah. Vestnik Rybinskoj gosudarstvennoj aviacionnoj tekhnologicheskoy akademii im. P.A. Solov'eva. 2017; 43(4):61–68. (In Russ).
4. Chernyh A.A., Sharapov A.I., Peshkova A.V. Akusticheskie processy v gazokapel'nykh sredakh. Vestnik Tambovskogo gosudarstvennogo tekhnicheskogo universiteta. 2018; 24(2):281–286 (In Russ).
5. Nigmatulin R.I., Aganin A.A., Toporkov D.Yu., Il'gamov M.A. Obrazovanie skhodyashchih udarnykh voln v puzyr'ke pri ego razrushenii. DOKLADY FIZIKI. 2014; 59(9): 431-435. (In Russ). doi: 10.1134/S1028335814090109
6. Nigmatullin R.I., Aganin A.A., Ilgamov M.A. et al. Sil'noe szhatie para v kavitacionnykh puzyr'kakh v vode i acetone. Vestnik Bashkirskogo universiteta. 2017; 22 (3): 580-585. (In Russ).
7. Chernykh A.A., Peshkova A.V., Sharapov A.I. Osnovnye polozheniya teorii akustiki dvufaznykh sred 5 Mezhdunarodnaya molodezhnaya nauchnaya konferenciya. 2018. Pp 302-305. (In Russ).
8. Gubarev V.Ya. Skorost' zvuka v gazozhidkostnykh sredakh. V sbornike: Energoberezhenie i effektivnost'

#### Литература

1. Черных А.А., Пешкова А.В. Начальные сведения о теории распространения звука в двухкомпонентных смесях // Труды 3-й Международной научной конференции студентов и молодых ученых. «Молодежь и системная модернизация страны»; 22-23 мая 2018., Курск, 2018. С. 207–211.
2. Накоряков В.Е., Покусаев Б.Г., Шрейбер И.Р. Распространение волн в газовых и парожидкостных средах. Новосибирск ИТФ СО АН СССР, 1983. 237 с.
3. Губарев В.Я. Газожидкостные течения в соплах. // Вестник Рыбинской государственной авиационной технологической академии им. П.А. Соловьева. 2017. № 4 (43). С. 61–68.
4. Черных А.А., Шарапов А.И., Пешкова А.В. Акустические процессы в газокapel'ных средах. // Вестник Тамбовского государственного технического университета. 2018. № 2(24). С. 281–286.
5. Нигматулин Р.И., Аганин А.А., Топорков Д.Ю., Ильгамов М.А. Образование сходящихся ударных волн в пузырьке при его разрушении. ДОКЛАДЫ ФИЗИКИ. 2014. Т. 59. № 9. С. 431–435.
6. Нигматулин Р.И., Аганин А.А., Ильгамов М.А., Топорков Д.Ю. Сильное сжатие пара в кавитационных пузырьках в воде и ацетоне. // Вестник Башкирского университета. 2017. № 3. (22) С. 580–585.
7. Черных А.А., Пешкова А.В., Шарапов А.И. Основные положения теории акустики двухфазных сред // Труды 5-й Международной молодежной научной конференции. 2018. С. 302–305.
8. Губарев В.Я. Скорость звука в газожидкостных средах. В сборнике: Энергосбережение и

- v tekhnicheskikh sistema ..Mezhdunarodnaya nauchno-tekhnicheskaya konferenciya studentov, molodyh uchenyh i specialistov. Russia 2017.10-12 Jul; Tambov, Russia. Tambovskij gosudarstvennyj tekhnicheskij universitet.(In Russ).
- 9.Nigmatulin RI., Bolotnova RH. Shirokoe uravnenie sostoyaniya vody i para v uproshchennoj forme. *Teplofizika vysokih temperatur*.2011;49(2):303-306. (In Russ).
- 10.Pokusaev BG., Tairov EA., Vasil'ev SA. Nizkochastotnye volny davleniya v parozhidkostnoj srede s fiksimovannym sloem sfericheskikh chastic. *Akusticheskaya fizika*. 2010; 56(3):306-312.(In Russ).
11. Fedotovskiy VS., Prokhorov YuP., Vereshchagina TN.Dinamicheskaya plotnost' i skorost' rasprostraneniya voln davleniya v dispersnyh sredah // *Teploenergetika* 2001; 48(3): 70–74.(In Russ).
12. Doncov VE., Nakoryakov VE. Processy obrazovaniya i rastvoreniya gidratov za udarnoj volnoj v zhidkosti, sodержashchej puzyr'ki gaza. *Zhurnal prikladnoj mekhaniki i tekhnicheskoy fiziki*. 2009; 50(2):318-326. (In Russ).
13. Shagapov VI. Vliyanie teplomassobmennyh processov mezhdu fazami na rasprostranenie malyh vozmushchenij v pene. *Teplofizika vysokih temperatur*. // *Vestnik Bashkirskogo Universiteta*. 1985; 23(1):126-132.(In Russ).
14. Chernyh AA., Gubarev VYa. Rasprostranenie zvukovyh voln v dvuhfaznyh sredah. 5 Mezhdunarodnoj nauchno-tekhnicheskoy konferencii studentov, molodyh uchyonih i specialistov «Energosberezhenie i effektivnost' v tekhnicheskikh sistemah». 2018. pp. 272–273. (In Russ).
15. Nakoryakov VE., Doncov VE., Pokusaev BG. Rasprostranenie voln davleniya v zhidkosti s tverdymi chasticami i puzyr'kami gaza. *Inzhenernaya teplofizika*. 1994 ; 4(2):173–188.(In Russ).
16. Nigmatulin RI., Aganin AA., Toporkov DYU., Ilgamov MA. The formation of converging shock waves in a bubble during its compression. *Reports of the Academy of Sciences*. 2014. 458(3):282.(In Russ). doi: 10.7868/S0869565214270115
17. Nigmatulin RI., Gubajdullin DA., Fedorov YuV. Akusticheskie volny raznoj geometrii v polidispersnyh puzyr'kovykh zhidkostyah. *Teoriya i eksperiment*. 2013;450(6): 665.(In Russ).
18. Nigmatulin RI., Aganin AA., Ilgamov MA., Toporkov DYU. Evolyuciya vozmushchenij sfericheskoy formy kavitacionnogo puzyr'ka. *Uchenye zapiski Kazanskogo universiteta*.. 2014; 156
- эффективность в технических система // Труды Международной научно-технической конференции студентов, молодых ученых и специалистов. Тамбовский государственный технический университет. 10-12 июля 2017.Тамбов.
9. Нигматулин Р.И., Болотнова Р.Х. Широкое уравнение состояния воды и пара в упрощенной форме.// *Теплофизика высоких температур* .2011. Т.49. № 2 С. 303-306.
10. Покусаев Б.Г., Таиров Е.А., Васильев С.А. Низкочастотные волны давления в парожидкостной среде с фиксированным слоем сферических частиц. // *Акустическая физика*.. 2010.56. № 3(56). С. 306-312.
11. Федотовский В.С., Прохоров Ю.П., Верещагина Т.Н. Динамическая плотность и скорость распространения волн давления в дисперсных средах // *Теплоэнергетика*. 2001. № 3(48). С. 70–74.
12. Донцов В.Е., Накоряков В.Е., Донцов Е.В. Процессы образования и растворения гидратов за ударной волной в жидкости, содержащей пузырьки газа // *Журнал прикладной механики и технической физики*. 2009; № 2(50) .С. 318-326.
13. Шагапов В.И. Влияние тепломассобменных процессов между фазами на распространение малых возмущений в пене. *Теплофизика высоких температур*. // *Вестник Башкирского Университета*.. 1985 Т.23.№ 1. С.126-132.
14. Черных А.А., Губарев В.Я. Распространение звуковых волн в двухфазных средах// Труды 5 Международной научно-технической конференции студентов, молодых учёных и специалистов «Энергосбережение и эффективность в технических системах». 2018. С. 272-273.
15. Накоряков В. Е., Донцов В. Е., Покусаев Б. Г. Распространение волн давления в жидкости с твердыми частицами и пузырьками газа // *Инженерная теплофизика*. 1994. Т.4.№ 2. С. 173–188.
16. Нигматулин Р.И., Аганин А.А., Топорков Д.Ю., Ильгамов М.А. Образование сходящихся ударных волн в пузырьке при его сжатии. Доклады академии наук. 2014.Т.458. № 3. С.282.
17. Нигматулин Р.И., Губайдуллин Д.А., Федоров Ю.В. Акустические волны разной геометрии в полидисперсных пузырьковых жидкостях. Теория и эксперимент. 2014. Т.450. № 6 . С.665.
18. Нигматулин Р.И., Аганин А.А., Ильгамов

(1): 79-108.(In Russ).

М.А., Топорков Д.Ю. Эволюция возмущений сферической формы кавитационного пузырька. Ученые записки Казанского университета. 2014.Т.156. № 1. С. 79-108.

**Authors of the publication**

*Aleksei I. Sharapov* – Department of industrial heat power engineering, Lipetsk State Technical University.

*Anton A. Chernykh* – Department of industrial heat power engineering, Lipetsk State Technical University.

*Anastasia V. Peshkova* – Department of industrial heat power engineering, Lipetsk State Technical University.

***Received***

***April 09, 2019***