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DETERMINATION OF LOCAL HEAT TRANSFER COEFFICIENTS AT THE ENTRANCE REGION OF STREAMLINED BODIES

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Abstract: *Prandtl's two-layer model of the turbulent boundary layer is considered and the expression obtained through the use of the model is applied to calculate the heat transfer coefficient, calculations for which agree well with experimental data on mean values of the coefficients for various bodies. Determination of parameters of this expression is shown for the case of calculating local heat transfer coefficients in the entrance regions of the channels. The main parameters are dynamic velocity, dimensionless thickness of the boundary layer and dimensionless thickness of the viscous sublayer. Based on the power-law and logarithmic velocity profiles, expressions are obtained for calculating the dimensionless parameters of the turbulent boundary layer. A satisfactory agreement of the results of calculations of local heat transfer coefficients for the flow over a flat plate and the pipe flow is shown. The presented approach represents a theoretical basis for modeling the local heat transfer for bodies of more complex shapes, if the friction coefficients are known.*

Keywords: *local heat transfer, boundary layer, turbulence, entry region.*

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Introduction

Solving the problems of mathematical modeling and improving the efficiency of heat transfer processes represent an important task, which is relevant for practically all industries as well as for power engineering [1–4]. Several monographs and textbooks have been published in recent years in this field [5–7].

In the course of solution of the problems of mathematical modeling of heat transfer processes, in addition to the average heat transfer coefficients in the flow around various bodies of relatively small spills (for example, at the entrance regions of short channels), the calculation of local heat transfer coefficients must be performed. The hydrodynamic stabilization of the boundary layer in the channels takes place at the entrance region, the length of which ranges from 20 to 50 pipe diameters, depending on the Reynolds number.

The purpose of the present work is to present examples of calculations of local heat transfer coefficients for flows over a flat plate and in a pipe based on the application of Prandtl's boundary layer model.

Prandtl's two-layer model of the turbulent boundary layer is the simplest model from the standpoint of its mathematical description; however, it gives results for the transfer coefficients, which are only slightly different from the results obtained via the more complex models (Karman, Deissler, Levich, Hanratty, Owen, Van Driest and others).

Based on application of Prandtl's model, an expression for the heat transfer coefficient was obtained in the form [8, 9]:

$$\alpha = \frac{\rho c_p u_*}{\Pr^m \left[R_1 + \frac{1}{\chi} \ln\left(R_\delta / R_1 \right) \right]}, \tag{1}$$

where α is heat transfer coefficient, W/m^2·K; Pr is Prandtl number; u_* is dynamic velocity, m/s; R_1 is dimensionless thickness of the viscous sublayer (for fully-developed flat plate flow $R_1 = 11,6$); $R_\delta = u_* \delta / \nu$ is dimensionless thickness of the boundary layer; δ is thickness of the boundary layer, m; ν is kinematic viscosity, m^2/s; $\chi = 0,4$ is turbulence constant; ρ is density of the medium, kg/m^3; c_p is specific heat capacity of the medium, J/kg·K.

Expression (1) gives satisfactory results of calculations for average heat transfer coefficients under various flow conditions in the channels [8, 9].

The use of expression (1) for calculating local heat transfer coefficients is discussed below.

Local heat transfer from a plate

At first, let us consider an example of flow over a flat plate in the turbulent stationary mode. For this case, we need to determine the main parameters in expression (1) depending on the distance of the free-stream flow in the longitudinal direction of the plate. Then for the flat plate flow (m=0,57), we can write

$$\alpha_x = \frac{\rho c_p u_{*x}}{\Pr^{0,57} \left[R_{1x} + \frac{1}{\chi} \ln\left(R_{\delta x} / R_{1x} \right) \right]} \tag{2}$$

Dynamic velocity is expressed through the local friction factor of the plate C_{fx}:

$$u_{*x} = u_\infty \sqrt{C_{fx} / 2}, \tag{3}$$

where u_∞ is external-flow velocity, m/s.

For the flat plate flow at $\mathrm{Re}_x = u_\infty x / \nu$ in the range from 10^5 to 10^6, $C_{fx} = 0,058 / \mathrm{Re}_x^{0,2}$, where Re_x is Reynolds number; x is longitudinal coordinate, m.

An expression for the local value of thickness of the turbulent boundary layer on the plate, as is known, has the form

$$\delta = \frac{0,37x}{\mathrm{Re}_x^{0,2}}. \tag{4}$$

Then an expression for the value of $R_{\delta x}$ can be obtained from (3) and (4) in the form:

$$R_{\delta x} = 0,37 \, \mathrm{Re}_x^{0,8} \sqrt{C_{f\,x} / 2}. \tag{5}$$

Dimensionless thickness of the viscous sublayer R_{1x} can be expressed as a function of the coordinate, which can be found from the power-law velocity profile on the plate:

$$\frac{u}{u_*} = C(n) \left(\frac{y u_*}{\nu} \right)^{1/n} = C(n) \left(y^+ \right)^{1/n}, \tag{6}$$

where for $40 < y^+ < 700$, $C(n) = 8,74$; $n = 7$.

In the viscous sublayer, the velocity profile is described by the linear function:

$$\frac{u}{u_*} = \frac{u_* y}{\nu}. \qquad (7)$$

At the boundary of the viscous sublayer, functions (6) and (7) give the identical value

$$R_1 = \frac{u_* \delta_1}{\nu} = C(n)\left(\frac{u_* \delta_1}{\nu}\right)^{1/n}. \qquad (8)$$

The value $C(n)$ can be determined from the velocity profile (6); at $y = \delta$ and $u = u_\infty$, we have

$$\frac{u_\infty}{u_*} = C(n) R_\delta^{-1/n}. \qquad (9)$$

Then

$$C(n) = \frac{u_\infty}{u_*} R_\delta^{-1/n}. \qquad (10)$$

As a result, from (8)–(10) we obtain for *n*=7 a local value, where $u_{*x} = u_\infty \sqrt{C_{fx}/2}$ (3); thus, we have

$$R_{1x} = \left(\frac{2}{C_{fx}}\right)^{7/12} R_{\delta x}^{-1/6}. \qquad (11)$$

Calculations reveal that the value obtained via (11) practically does not change and is equal to $R_{1x} = 12,4$, which is close to the value $R_1 = 11,6$ provided by the boundary layer theory. It is likely that a small discrepancy is due to an error of the approximation of the velocity profile by a power-law function. Therefore, we can adopt $R_{1x} = R_1 = 11,6$.

A velocity profile in the turbulent region of the boundary layer is also described by a logarithmic function having the form

$$\frac{u}{u_*} = 2,5 \ln \frac{u_* y}{\nu} + 5,5. \qquad (12)$$

At the boundary of the viscous sublayer, the linear velocity profile and the logarithmic one take the same value, i.e. at $y = \delta_1$: $u_1 / u_* = R_1$ and

$$R_1 = 2,5 \ln R_1 + 5,5. \qquad (13)$$

This expression leads to the constant value $R_1 = 11,63$.

The local heat transfer coefficient on the plate is calculated via the known criterial expression [1]

$$\mathrm{Nu}_x = 0,03 \mathrm{Re}_x^{0,8} \mathrm{Pr}^{0,43}, \qquad (14)$$

where $\mathrm{Nu}_x = \alpha_x x / \lambda$ is local Nusselt number; λ is thermal conductivity of the medium, W/m K.

Expression (2) can be written in a dimensionless form with u_{*x} determined from (3)

$$\mathrm{Nu}_x = \frac{\mathrm{Re}_x \sqrt{C_{fx}/2}\, \mathrm{Pr}^{0,43}}{11,6 + 2,5 \ln\left(R_{\delta x}/11,6\right)}. \qquad (15)$$

From calculations via (14) and (15) at $\mathrm{Re}_x = 2 \cdot 10^5$ in line with expression (14), we have $\mathrm{Nu}_x = 513,6$, and line with (15), we have $\mathrm{Nu}_x = 502,5$ (at $\mathrm{Pr} = 1$).

The deviation is around 3%. At $\mathrm{Re}_x = 10^6$, accordingly we obtain $\mathrm{Nu}_x = 1861$ and $\mathrm{Nu}_x = 1882$, and the deviation is around 2%.

Thus, adequacy of expression (15) for Nu_x with parameters (5) and $R_1 = 11{,}6$ is proved for modeling of local heat transfer on the plate under turbulent flow conditions.

Heat transfer at the entrance region of the pipe

At turbulent motion of a single-phase flow in a pipe, length of the hydrodynamic stabilization region $l_{CT} < 50d$, where d is pipe diameter [1, 10]. At the entrance region, the flow velocity on the channel axis changes from an average inlet value u_{av} to a value $u_{\max}$ present beyond the hydrodynamic stabilization region. Taking into account that the thickness of the turbulent boundary layer (4) depends on the longitudinal coordinate x as $\delta \sim x^{4/5}$, the flow velocity on the axis can be approximately determined from the expression

$$u_{\max(x)} = u_{av} + u_{m(x)} \left(x / l_{CT} \right)^{4/5}, \ \left(x \le l_{CT} \right), \qquad (16)$$

where at x=0 we have $u_{\max} = u_{av}$ (entrance to the pipe); at $x = l_{CT}$, $u_{\max} = u_{av} + u_m$, where $u_m = 4u_*$; at $0 < x < l_{CT}$, $u_m = 4u_{*(x)}$ is velocity on the axis, m/s.

In the literature, there is no function for the friction coefficient for the entrance region of the pipe; therefore, for the first approximation at $x < l_{CT}$, we make use of the expression for the flat plate flow $C_{f\,x}$ and the dynamic velocity in formula (3).

The value of $R_{\delta x}$ is calculated via formula (5), where the Reynolds number is calculated through the velocity from (16), similarly to calculation of $C_{f\,x}$.

In addition, from the logarithmic profile (12), one can obtain a local value of $R_{\delta x}$ for the entrance region of the pipe. For $u = u_\infty$ and $y = \delta$, we have

$$R_{\delta x} = \exp\left[0{,}4 \left(\frac{u_{av} + 4u_{*x}}{u_{*x}} \right) - 5{,}5 \right]. \qquad (17)$$

Length of the hydrodynamic stabilization region in a circular pipe can be approximately estimated from expression (4) at $\delta \approx R$ and $u_\infty \approx 1{,}15 u_{cp}$. We obtain

$$l_{CT} = \left(\frac{R}{0{,}37} \right)^{5/4} \left(\frac{1{,}15 u_{av}}{\nu} \right)^{1/4}, \qquad (18)$$

where R is pipe radius, m.

The calculation shows that the ratio of the local heat transfer coefficient α_x (2) to the average one α at $x/d = 1{,}0$ becomes $\alpha_x / \alpha = 1{,}35$ ($\mathrm{Re}_d = 5 \cdot 10^4$). In monographs [1, 7, 10], the value $\alpha_x / \alpha = 1{,}34$ was given. The calculations of α_x / α are in satisfactory agreement with the known corrections, which take into account the entrance region in the pipe at different Reynolds numbers and, accordingly, length of the entrance region (18).

Conclusions

The use of an expression for heat transfer coefficient obtained earlier by the authors via Prandtl’s model is considered for the case of local heat transfer at the entrance regions of the channels. The local boundary layer parameters for flows over a flat plate and in a pipe are determined. An agreement of the results of calculations for local heat transfer coefficients versus known results is shown. The presented approach is a theoretical basis for modeling local heat transfer for bodies having different geometries of streamlined surfaces.

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