PHYSICS

UDC 593.3 DOI:10.30724/1998-9903-2019-21-3-4-116-126

THE STOCHASTIC FORMULATION OF THE STEPHAN ROBLEM IN HYPERBOLIC REPRESENTATION

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Abstract: *The presented work offers the stochastic description of the Stephan problem on the basis of a deterministic model in hyperbolic representation. This description is based on the generalized Fokker-Planck-Kolmogorov equation. The basic thesis of this work is that the determined equalizations and their solutions are the average values of the stochastic Stephan problem model. This work considers the problem of phase transition front deformation. The research is performed using the introduced position of stability on solutions dispersion for average values. The conclusion of this study is that Markov diffusion coefficient results in a significant distortion of the originally flat front of the phase boundary.*

Keywords: *Stefan problem, hyperbolic equation of heat conduction, generalized Fokker-Planck-Kolmogorov equation, stability of solutions of differential equations, deformation of phase transition front.*

For citation: Kartashov E.M., Soloviev I.A. The stochastic formulation of the Stephan's roblem in hyperbolic representation. *Proceedings of the higher educational institutions. ENERGY SECTOR PROBLEMS*. 2019; 21(3-4):116-126. (In Russ). doi:10.30724/1998-9903-2019-21-3-4-116-126.

Introduction

We list the problems related to deterministic description of thermal conductivity during phase transformations. The first problem concerning deterministic Fourier model and the classical Stefan problem [1] is in infinity of propagation velocity of initial temperature perturbations, as well as in infinity of initial velocity of the phase boundary movement. The second problem is that the deterministic model did not describe the time dependent deformation of the originally flat form of the phase transition.

The study of effects in fast-flowing manifestations of heat conduction goes back to the works of Maxwell–Cattaneo–Lykov described in [2, 3]. These papers describe the generalized Fourier law:

$$\partial\overline{q}\left(x,y,z,t\right)/\partial x=-\lambda\,\mathrm{grad}T\left(x,y,z,t\right)-\tau_r\partial q(x,y,z,t)/\partial t\,, \tag{1}$$

which takes into account the final rate of heat propagation. Here (x,y,z) are the spatial coordinates; t is time coordinate; $\bar{q}$ is heat flow density; T is temperature; λ is coefficient of thermal conductivity; a is coefficient of thermal diffusivity; τ_r is heat flow relaxation time, which is connected with the rate of heat propagation v_T by the ratio $v_T = \sqrt{a/\tau_r}$. Expression (1) has a simple physical meaning: when a temperature gradient occurs, it takes some time to establish the heat flow, when $\mathrm{grad}T = 0$ the heat flow does not disappear instantaneously, but decays with the relaxation time. After analyzing the generalized problem of heat conduction for a half-space, the boundary temperature of which changes at the initial moment of time by a certain amount, remaining further constant, A.V. Lykov gave a substantiation of the physical meaning of the final velocity of heat propagation, which is a time derivative of the depth of heat penetration. Expression (1) is reduced to a deterministic transfer equation of the hyperbolic type:

$$\begin{gathered}\partial T(M,t)/\partial t = a\Delta T(M,t) - \tau_r \partial^2 T(M,t)/\partial t^2 + \\ +(\tau_r/(c\rho))\left[\partial F(M,t)/\partial t + (1/\tau_r)F(M,t)\right]\end{gathered}, \quad M \in D, \quad t > 0 \qquad (2)$$

and the corresponding boundary problems of heat conduction for equation (2) in *generalized form*. Generalized transfer problems are significantly different from the classical ones, being more complex when finding analytical solutions to these problems. This results in very insignificant progress in finding exact analytical solutions of boundary problems for equation (2). As it will be shown below, these analytical values are involved in dispersion formation. At the same time, it is necessary to note considerable attention to the Stefan problem from physicists studying the impact of laser radiation on matter [5–9].

In this paper, we present a description of the random behavior of highly non-stationary heat conduction using the generalized Fokker-Planck-Kolmogorov equation (hereinafter FPK) for probability density (hereinafter PD), from which the problem statements for the hyperbolic heat equation are obtained. The novelty lies in the fact that stochastic formulations of the Stefan type for hyperbolic thermal conductivity have so far been absent. The bibliography and the main ideas of such a stochastic description of heat conduction problems are presented in [2, 3]. The basic statement is as follows: the solution of a deterministic problem is the average value of its stochastic analog. The urgency of the problem lies in the fact that the deterministic task does not allow to identify the features that arise as a result of taking into account the random external influence on the described phenomena. In [3], it was shown how dispersion can significantly change the understanding of the conclusions which follow from the analysis of deterministic solutions to the problems posed. The study of the temporal behavior of dispersion made it possible to obtain the effect of dispersion decrease described in [3] at initial times. This effect makes it possible to plan experiments related to phase transitions. Here we will consider analysis of dispersion with a powerful pulsed thermal impact on the substance.

Stochastic model of the Stefan problem, which takes into account the finite rate of heat transfer

A stochastic model of the classical Stefan problem, based on the generalized FPK equation connected with the parabolic heat equation, was proposed in [3]. We give the formulation of this problem. First, we introduce the notations for the stochastic description of the Stefan problem, taking into account the finite rate of heat transfer. We denote by $P_1(x,t,\Omega)$ the probability density in the spatial domain $0 \le x \le M_y^{(1)}(t)$ (solid phase), and by $P_2(x,t,\Omega)$ the probability density in the spatial domain $x \ge M_y^{(1)}(t)$ (liquid phase), where $M_y^{(1)}(t)$ is average velocity of the interface; t is time; x is spatial coordinate; Ω is random characteristic of the temperature field in both areas. We also denote the average values as $M_{Ti}^{(1)}(x,t) = \int_{-\infty}^{+\infty} \Omega P_i(x,t,\Omega)d\Omega \quad (i=1,2)$; the second order moment as $M_{Ti}^{(2)}(x,t)=\int_{-\infty}^{+\infty}\Omega P_i(x,t,\Omega)d\Omega$, $(i=1,2)$; dispersion as $D_{Ti}(x,t)=\int_{+\infty}^{+\infty}\Omega^2 P_i(x,t,\Omega)d\Omega-(\int_{-\infty}^{+\infty}\Omega P_i(x,t,\Omega)d\Omega)^2$ $(i=1,2)$. We also introduce the notation for PD, describing random processes occurring at the phase interface: $P_y(t,\Theta)$, here Θ is the random characteristic of processes which determine the behavior of phase interface. We denote the mean value of the time dependence of the phase boundary motion law by $M_y^{(1)}(t)=\int_{-\infty}^{+\infty}\Theta P_y(t,\Theta)d\Theta$, and dispersion by $D_y(t)=\int_{-\infty}^{+\infty}\Theta^2 P_y(t,\Theta)d\Theta-(\int_{-\infty}^{+\infty}\Theta P_y(t,\Theta)d\Theta)^2$. The Markov diffusion coefficient for random manifestations at the boundary is denoted as B_Θ. The FPK equation for PD describing a random thermal field in the solid state region is

$$\partial P_1(x,t,\Omega)/\partial t=-\partial(A_1(x,t,\Omega)P_1(t,x,\Omega))/\partial\Omega+0,5B_\Omega\partial^2 P_1(x,t,\Omega)/\partial\Omega^2,$$
$$0<x<M_y^{(1)}(t),\ \ t>0,\ \Omega\in(-\infty<+\infty). \quad (3)$$

Here
$A_1(x,t,\Omega)=\Omega(a_1\partial^2 M_{T1}^{(1)}(x,t)/\partial x^2-\tau_r\partial^2 M_{T1}^{(1)}(x,t)/\partial t^2)/M_{T1}^{(1)}(x,t)$.
The drift coefficient has a similar form $A_2(x,t,C)$:

$$\partial P_2(x,t,\Omega)/\partial t=-\partial(A_2(x,t,\Omega)P_2(x,t,\Omega))/\partial\Omega+0,5B_\Omega\partial^2 P_2(x,t,\Omega)/\partial\Omega^2,$$
$$M_y^{(1)}(t)<x<+\infty,\ t>0,\ \Omega\in(-\infty<+\infty),$$

where B_Ω is the Markov diffusion coefficient; a_i $(i=1,2)$ are thermal diffusivity coefficients in the regions corresponding to the normalization condition $\int_{-\infty}^{+\infty}P_i(x,t,\Omega)d\Omega=1,\ t\geq 0,\ x\in(0,+\infty)$ and conditions at infinity $P_i(x,t,\pm\infty)=0$.

Of course, the classical Stefan problem cannot claim to describe the fast-flowing effects. There is a difference between the slow-flowing formation of ice in natural phenomena and melting, when a substance undergoes a short-term thermal shock effect. Although, we note, even in such a stochastic formulation of the classical Stefan problem, the effect of the “strange” dispersion behavior was revealed in [5]. The essence of the effect is as follows. At the initial time moment, the regular component of dispersion is zero and up to the time moment $t=1/e$ (time units) it decreases, reaching a minimum equal to $\min\operatorname{Re}g(1/e)=-B_\Theta/e$. This means that at this time interval there is resistance to changing the shape of the phase transition front. We called this phenomenon *the effect of striving to preserve the original form of the phase transition front*. After the time moment $t=1/e$ (time units), this resistance weakens and stops at the time moment $t=1$ (time units), when $\min\operatorname{Re}g(1)=0$. After this, the phase of active distortion of phase transition front begins.

This seemingly insignificant effect, due to the smallness of the Markov diffusion coefficient B_Θ and time $t=1/e$ (time units), may be important for thin technological processes, when it is required to preserve the initial planar shape of either the grown crystal or the planar shape of a solid body as long as possible under other thermal impacts, not necessarily having the nature of phase transformations, for example when applying nanocoatings. Knowing how the dispersion D_y changes over time, one should stepwisely conduct the technological process, starting from time moment $t=0$ up to $t=1/e$ (time units). Then make a stop, then start again, and periodically repeat this procedure many times. In this case, unnecessary random effects will be neutralized, which contribute to the distortion of flat shape of a solid body exposed to external and internal random effects.

In this paper, we study one of the variants of stochastic formulation for the PD problem of the Stefan type.

We present the following problem statement for PD. We assume that a powerful thermal impulse affects a substance when its duration $1/\alpha$ is comparable with the time of thermal relaxation τ_r. We also assume that a fixed boundary is exposed to radiation with heat flow capacity equal to $q_0\exp[-\alpha t]$, and a heat flow of capacity equal to $q_1\exp[-\alpha t]$, where $q_1>q_0$. acts on a moving boundary. The stochastic formulation of the problem for PD on the fast-flowing process of heat conduction with a pulsed effect of powerful radiation on a substance has the following form:

$$\partial P(x,t,\Omega)/\partial t=-\partial(A_T(x,t,\Omega)P(t,x,\Omega))/\partial\Omega+0{,}5B_\Omega\partial^2P(x,t,\Omega)/\partial\Omega^2,$$
$$0<x<M_y^{(1)}(t),\ t>0,\ \Omega\in(-\infty<+\infty). \tag{4}$$

Here:

$$A_T(x,t,C)=\Omega P_T(x,t,\Omega)(a_1\partial^2M_T^{(1)}(x,t)/\partial x^2-\tau_r\partial^2M_T^{(1)}(x,t)/\partial t^2)/M_T^{(1)}(x,t) \tag{5}$$

$$P_T(0,t,\Omega)=\varphi_{T\text{гран}}(t,\Omega),\ \Omega\in(-\infty,+\infty),\ 0<t<+\infty. \tag{6}$$

$$-\lambda\partial P_T(x,t,\Omega)/\partial x\Big|_{x=M_y^{(1)}(t)}=$$
$$=-\partial(\Omega L\rho(dM_y^{(1)}(t)/dt+\tau_r d^2M_y^{(1)}(t)/dt^2+$$
$$+q_1\exp[-\alpha t])P_T(M_y^{(1)}(t),t,\Omega)/M_T^{(1)}(M_y^{(1)}(t),t))/\partial\Omega+$$
$$+0{,}5B_\Theta\partial^2P_T(M_y^{(1)}(t)),t,\Omega)/\partial\Omega^2,\ x=M_y^{(1)}(t),\ t>0,\ \Omega\in(-\infty;+\infty). \tag{7}$$

$$P_T(x,0,\Omega)=\varphi_{Tini}(x,\Omega),\ 0\le x\le M_y^{(1)}(0),\ \Omega\in(-\infty,+\infty). \tag{8}$$

$$\partial P_T(x,t,\Omega)/\partial t\big|_{t=0}=0,\ t\ge 0,\ \Omega\in(-\infty,+\infty). \tag{9}$$

$$M_y(0)=M_{y0}=\text{const}. \tag{10}$$

$$dM_y(t)/dt\big|_{t=0}=\tilde{M}_{y0}=\text{const}. \tag{11}$$

Comparative analysis of the mathematical expectations of the hyperbolic and parabolic models of the Stefan problem

The task for the mathematical expectation corresponding to the problem (4) - (11) has the following form [8]:

$$\tau_r\partial^2M_T^{(1)}(x,t)/\partial t^2+\partial M_T^{(1)}(x,t)/\partial x=a\partial^2M_T^{(1)}(x,t)/\partial x^2,$$
$$0<x<M_y^{(1)}(t),\ t>0. \tag{12}$$

$$M_T^{(1)}(0,t)=M_{T0}^{(1)}(0,t),\ t>0. \tag{13}$$

$$\lambda\partial M_T^{(1)}(0,t)/\partial x=q_0\exp[-\alpha t],\ t>0. \tag{14}$$

$$-\lambda\partial M_T^{(1)}(M_y^{(1)},t)/\partial x=\tau_r L\rho d^2M_y^{(1)}/dt^2+L\rho dM_y^{(1)}/dt+q_1\exp[-\alpha t],\ t>0. \tag{15}$$

$$M_y^{(1)}(0)=M_{y0}^{(1)}=\text{const}. \tag{16}$$

$$dM_y^{(1)}(t)/dt\big|_{t=0}=0. \tag{17}$$

The following functions satisfy equations (12)–(17)

$$M_T^{(1)}(x,t)=M_{T0}^{(1)}(0,t)+$$
$$+q_0\exp[-t/\tau_r]\sin(x\sqrt{\alpha(1-\tau_r\alpha)/a})/(\lambda\sqrt{\alpha(1-\tau_r\alpha)/a}. \tag{18}$$

At $\alpha=1/\tau_r$ expression (18) is transformed to the form:

$$M_T^{(1)}(x,t)=M_{T0}^{(1)}(0,t)+q_0\exp[-t/\tau_r]x/\lambda. \tag{19}$$

By substituting (19) into (20), we obtain the expression

$$-\lambda\partial M_T^{(1)}(M_y^{(1)},t)/\partial x=d^2M_y^{(1)}/dt^2+dM_y^{(1)}/dt=$$
$$=-(1/(\tau_rL\rho))(q_1-q_0)\exp[-t/\tau_r],\ t>0. \tag{20}$$

The solution for equation (20) is:

$$M_y^{(1)}(t)=M_{y0}^{(1)}-(q_1-q_0)[1-\exp[-t/\tau_r](t/\tau_r+1)/(L\rho\tau_r). \tag{21}$$

These solutions were obtained and described in [8]. Direct substitution ensures that the solutions of the corresponding problems for mathematical expectations $M_{TT}^{(1)}(t,x)$ and $M_{yy}^{(1)}(t)$ based on the parabolic heat equation and the classical Fourier law have the following form:

$$M_{TT}^{(1)}=M_{TT0}^{(1)}(0,t)+q_0\sqrt{a/\alpha}\exp[-\alpha t]\sin(x\sqrt{\alpha/a})/\lambda. \tag{22}$$

$$M_{yy}^{(1)}(t)=M_{yy0}^{(1)}-2\sqrt{a/\alpha}\mathrm{arctg}\{(\sqrt{q_1^2-q_0^2}/(q_1+q_0))\times$$
$$\times\mathrm{tg}[\sqrt{q_1^2-q_0^2}/(2L\rho)(\sqrt{(a\alpha)^{-1}}(1-\exp(-\alpha t))]\}. \tag{23}$$

For a comparative analysis of solutions with hyperbolic and parabolic representations of the studied Stefan problem, it is necessary to transform solutions (22) - (23) at $\alpha=1/\tau_r$. We obtain:

$$M_{TT}^{(1)}=M_{TT0}^{(1)}(0,t)+q_0\sqrt{a\tau_r}\exp[-t/\tau_r]\sin(x/\sqrt{a\tau_r})/\lambda. \tag{24}$$

$$M_{yy}^{(1)}(t)=M_{yy0}^{(1)}-2\sqrt{a\tau_r}\mathrm{arctg}\{(\sqrt{q_1^2-q_0^2}/(q_1+q_0))\times$$
$$\times\mathrm{tg}[\sqrt{q_1^2-q_0^2}/(2L\rho)(\sqrt{\tau_r/a}(1-\exp(-t/\tau_r))]\}. \tag{25}$$

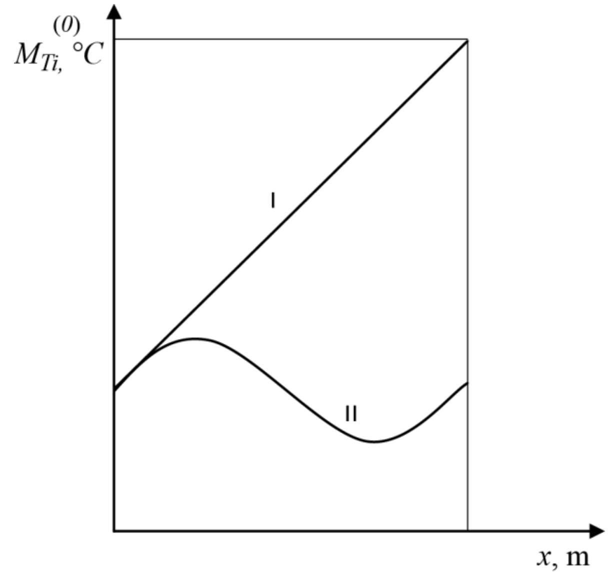

Fig.1. Relationship between temperature and the spatial coordinate:
i=1 is for parabolic model; i= 2 is for hyperbolic model

The curves for temperature dependences of $M_T^{(1)}(x,t)$ and $M_{TT}^{(1)}(x,t)$ are presented in Figs. 1 and 2. They show that solutions of problems for the hyperbolic and parabolic equations are not always of the same qualitative nature. If the function $M_{TT}^{(1)}(x,t)$ oscillates with a change in x, then the function $M_T^{(1)}(x,t)$ is strictly monotonic. For x values for which the sine in expression (24) is equal to 0, $M_{TT}^{(1)}(x,t)$ is constant in time, and $M_T^{(1)}(x,t)$ changes exponentially .

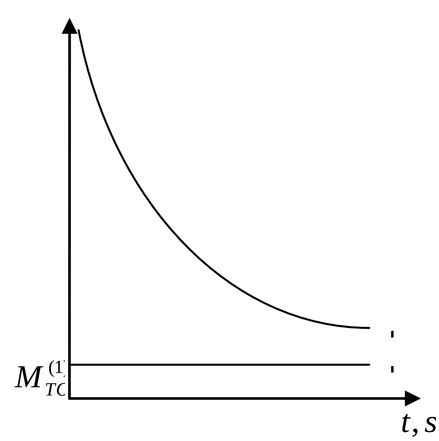

Fig.2. Relationship between temperature and time:
$i=1$ is for parabolic model; i= 2 is for hyperbolic model

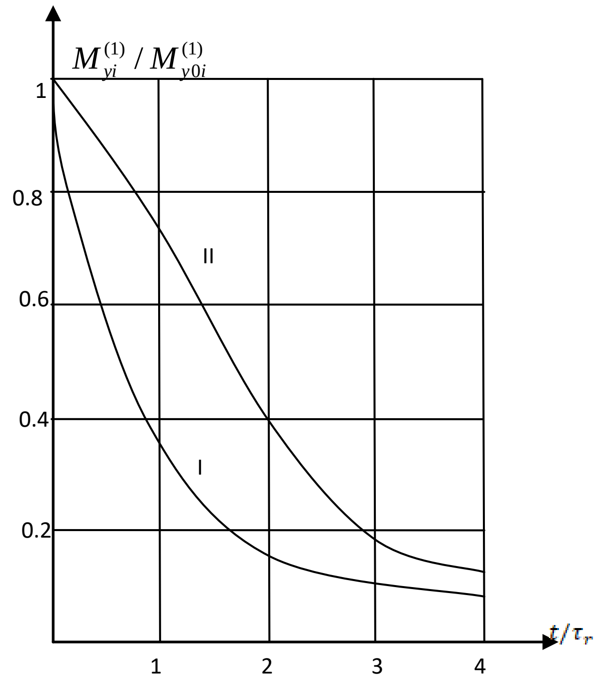

Fig. 3. Relationship between phase interface and time:
i=1 is for parabolic model; *i*= 2 is for hyperbolic model

$$\tau_r = 10^{-9} s; q_0 = 10^{-2} q_1; a = 10^{-5} m^2 / s$$

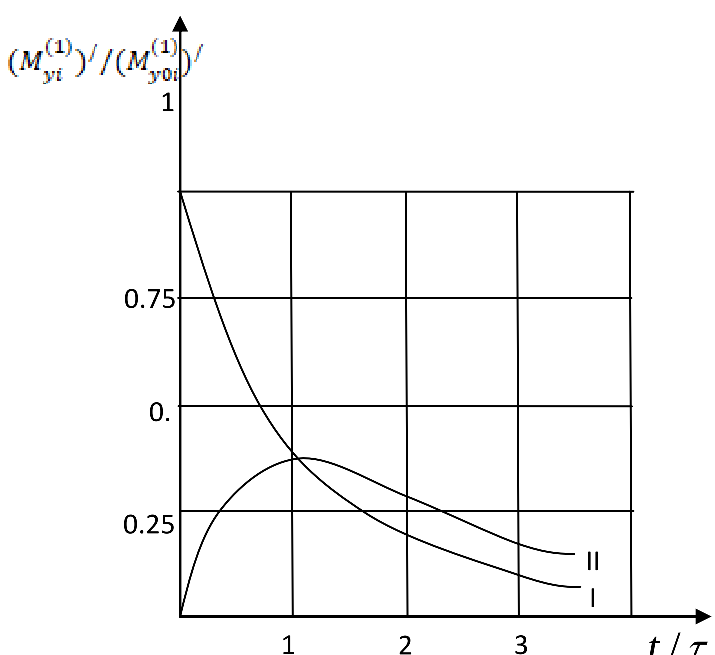

Fig. 4. Relationship between phase interface movement rate and time:
i=1 is for parabolic model; *i*= 2 is for hyperbolic model·

Figures 3 and 4 show behavior of motion laws for the phase transition front $M_y^{(1)}(t)$ and $M_{yy}^{(1)}(t)$. Figures 5 and 6 show behavior of motion laws for speeds $dM_y^{(1)}(t)/dt$ and $dM_{yy}^{(1)}(t)/dt$. A model with a parabolic equation predicts that the phase transition process begins at a maximum speed, which, we note, can be arbitrarily large, depending on the ratio between q_0 and q_1. According to the hyperbolic model, the phase transition front accelerates from zero speed to the maximum one not instantaneously, but during some time, determined by τ_r. The maximum value of $dM_y^{(1)}(t)/dt$ is limited above by the constant $\sqrt{a/\tau_r}$. Using relations (27)-(29), we can establish a relationship between the temperature at the front $M_T^{(1)}(M_y^{(1)}(t),t)$ and the kinematic characteristics of the front $dM_y^{(1)}(t)/dt$ and $d^2M_y^{(1)}(t)/dt^2$. For hyperbolic model we obtain

$$M_T^{(1)}(M_y^{(1)}(t),t) = M_{T0}^{(1)} + [(q_0L\rho)/(\lambda(q_1-q_0)][\tau_r L\rho d^2M_y/dt^2 + dM_y/dt)]M_y. \quad (26)$$

In a model with a parabolic equation, we get the following relation:

$$M_{yy}^{(1)}(t),t) = (M_{TT0}^{(1)} - (L\rho a)/\lambda) - a\tau_r L\rho d(\ln(M_{yy}^{(1)})/dt/\lambda\,. \quad (27)$$

Comparative analysis of dispersions of parabolic and hyperbolic models of the Stefan problem

The derivation of equations for PD and dispersion corresponding to the Stefan problems is not different from those proposed in [9]. Here we present the formulation of the problem for dispersion corresponding to the hyperbolic model of the investigated Stefan problem (20)–(25). This solution has the following form:

$$\partial D_T(x,t)/\partial t = [(2a\partial^2 M_T^{(1)}(x,t)/\partial x^2 - 2\tau_r\partial^2 M_T^{(1)}(x,t))/M_T^{(1)}(x,t)]D_T(x,t) + B_\Omega,$$
$$t>0,\ x\in(0,M_y^{(1)}(t)). \quad (28)$$

$$D_T(x,0) = D_{Tini}(x) = \delta_0^2(x)(M_{Tini}^{(1)})^2,\ x\in[0,M_y^{(1)}(0)]\,.$$

Dispersion of the phase transition front is described by the following equations:

$$\partial D_y(x,t)/\partial t = [(2a\partial^2 M_{T2}^{(1)}(x,t)/\partial x^2)/M_{T2}^{(1)}(x,t)]D_{T2}(x,t) + B_\Theta,\ t>0,\ x\in(M_y^{(1)}(t)),+\infty)\,. \quad (29)$$

$$D_T(x,0) = D_{Tini}(x) = \delta_0^2(x)(M_{Tini}^{(1)})^2,\ x\in[M_y^{(1)}(0),+\infty)\,. \quad (30)$$

Taking into account the fact that equations (12) - (17) are valid, equations (20) - (22) can be rewritten by replacing the right-hand side with the left-hand side:

$$\partial D_T(x,t)/\partial t = [\partial\ln(M_T^{(1)}(x,t))^2)/\partial t]D_T(x,t) + B_\Omega,\ \ t>0,\ x\in(0,M_y^{(1)}(t))\,.$$

From here we get solutions for dispersions in the form:

$$D_T(x,t) = [\int_0^t B_\Omega d\xi/(M_T^{(1)}(x,\xi))^2 + \delta_0^2(x)](M_T^{(1)}(x,t))^2,\ x\in(0,M_y^{(1)}(t)],\ t\ge 0. \quad (31)$$

$$D_y(t) = [\int_0^t B_\Theta dx/(M_y^{(1)}(x))^2 + \delta_0^2](M_y^{(1)}(t))^2,\ t\in[0,10] \quad (32)$$

For parabolic model we have:

$$D_{yy}(t) = [\int_0^t B_\Theta dx/(M_{yy}^{(1)}(x))^2 + \delta_0^2](M_{yy}^{(1)}(t))^2,\ t\in[0,10]\text{, where}$$

$$M_{yy}^{(1)}(t)=M_{yy0}^{(1)}-2\sqrt{a\tau_r}\operatorname{arctg}\{(\sqrt{q_1^2-q_0^2}\,/\,(q_1+q_0))\times \\ \times\operatorname{tg}[\sqrt{\tau_r/a}\cdot\sqrt{q_1^2-q_0^2}\cdot(1-\exp(-t)/(2L\rho)]\}. \quad (33)$$

Since the temperature mode during random impacts on a substance is unstable, to investigate the distortion of the phase transition front is of interest. This is also important because the initial moments of the motion velocities of this front differ greatly in the models under consideration.

So it turns out that the integrals $\int_0^t B_\Theta dx/(M_y^{(1)}(x))^2$ and $\int_0^t B_\Theta dx/(M_{yy}^{(1)}(x))^2$ cannot be resolved in quadratures, therefore it is proposed to find them numerically using the fourth-order Runge-Kutta method as a solution to the following Cauchy problem:

$$dz/dt=1/(M_y^{(1)}(t))^2,\ M_y^{(1)}(0)=0. \quad (34)$$

Further we present data for calculation of some material. The density is $\rho=2,7\cdot10^3$ kg/m^3. The heat of phase transition is $L=1{,}0449\cdot10^7$ J/kg. Heat conductivity coefficient is $\lambda=62$ W/(m·deg). The thermal diffusivity is $a=25.8\cdot10^{-6}$ m^2/s. Thermal relaxation time is $\tau_r=10^{-9}$ c. The initial (maximum) density of the incident flow on the moving boundary is $q_1=10^{11}$ W/m^2. The initial (maximum) density of the incident flow on a fixed boundary $q_0=10^9$ W/m^2. The initial position of the phase transition front for the hyperbolic model is $M_{y0}^{(1)}(0)=1$ m . The initial position of the phase transition front for the parabolic model is $M_{yy0}^{(1)}(0)=1$ m. The initial temperature for the hyperbolic model is $M_{T0}^{(1)}(x,0)=20^\circ$C. Initial temperature for the parabolic model $M_{TT0}^{(1)}(x,0)=20^\circ$C. Pulse time is $1/\alpha=\tau_r$,s. As for the Markov diffusion coefficients, their values are not presented in any reference book. In the present work, it is proposed to consider them of the same order with the thermal diffusivity coefficient $B_\Theta=10^{-4}$, $B_\Omega=10^{-4}$.

The calculation results are shown in Figs. 5 and 6.

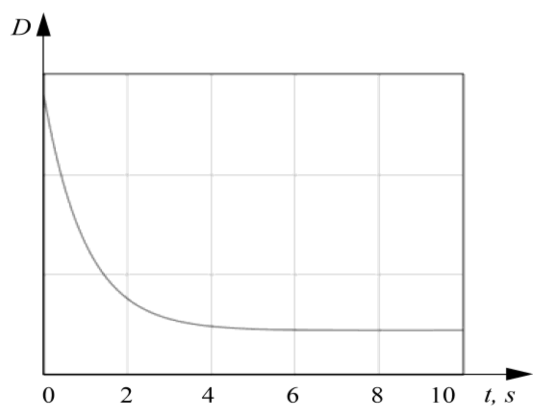

Fig. 5. The temporal behavior of the phase transition front dispersion for parabolic model

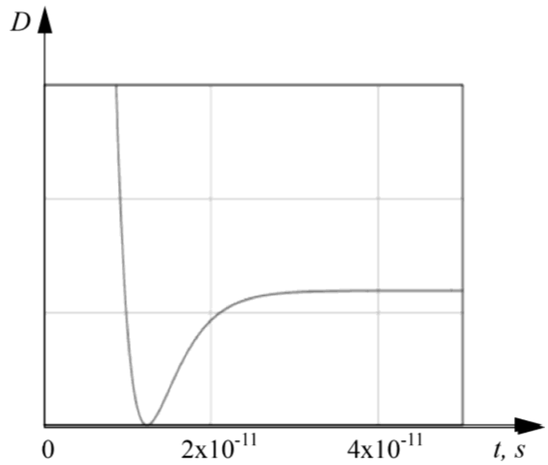

Fig. 6. The temporal behavior of the phase transition front dispersion for hyperbolic model

Further we discuss the results of the temporal behavior of the dispersion at the phase transition front. As in the case of the stochastic consideration of the Stefan problem, carried out in [5], the *effect of striving to preserve the original form of the phase transition front* is also observed here. How can we interpret the practical application of this phenomenon for the fast process of heat conduction? The relaxation time is very short, is it possible to benefit from identifying the time of occurrence of the smallest dispersion when distortion of the phase transition front shape is minimal? We try to answer this question. Despite the fact that the problem is considered here in a flat formulation, it is possible to carry out a similar quantitative and qualitative analysis for the spherical initial shape of the particle. Basing on fixing the moment of smallest dispersion, it is possible to calculate the pulse action time at which the smallest distortion of the phase transition front shape is observed, and as a result of planning the experiment, to obtain the desired configuration of the melted particle, possibly close to the initial one.

Conclusions

At present, a large mathematical apparatus has been accumulated, which describes many phenomena of a physical nature in a deterministic way. This apparatus requires its translation, figuratively speaking, into "stochastic language". In particular, in [2] one of the authors of this article (E.M. Kartashov) obtained numerous analytical solutions for original deterministic problems in thermal physics, thermo elasticity, etc., which have already been successfully used in practice. In recent papers of authors of this article, the possibilities of combining classical deterministic concepts of physical phenomena with stochastic ones are demonstrated. New effects can be discovered, in addition to the dispersion stability conditions described above and dispersion extreme properties. The practical application of the results of stochastic studies of motion laws for phase transition front to technological processes are of particular importance. This primarily concerns the determination of the time of melting and evaporation of microparticles at a powerful impact of pulsed laser radiation on matter.

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Received ***November, 23 2018***